Breaking the Mass Sheet Degeneracy with Gravitational Waves Interference in Lensed events

Based on arXiv:2104.07055

Paolo Cremonese

in collaboration with:

V. Salzano,

Institute of Physics, University of Szczecin, Szczecin, Poland

&

J.M. Ezquiaga,

Kavli Institute for Cosmological Physics and Enrico Fermi Institute, the University of Chicago, Chicago, USA

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Mass Sheet Degeneracy

Mass Sheet Degeneracy

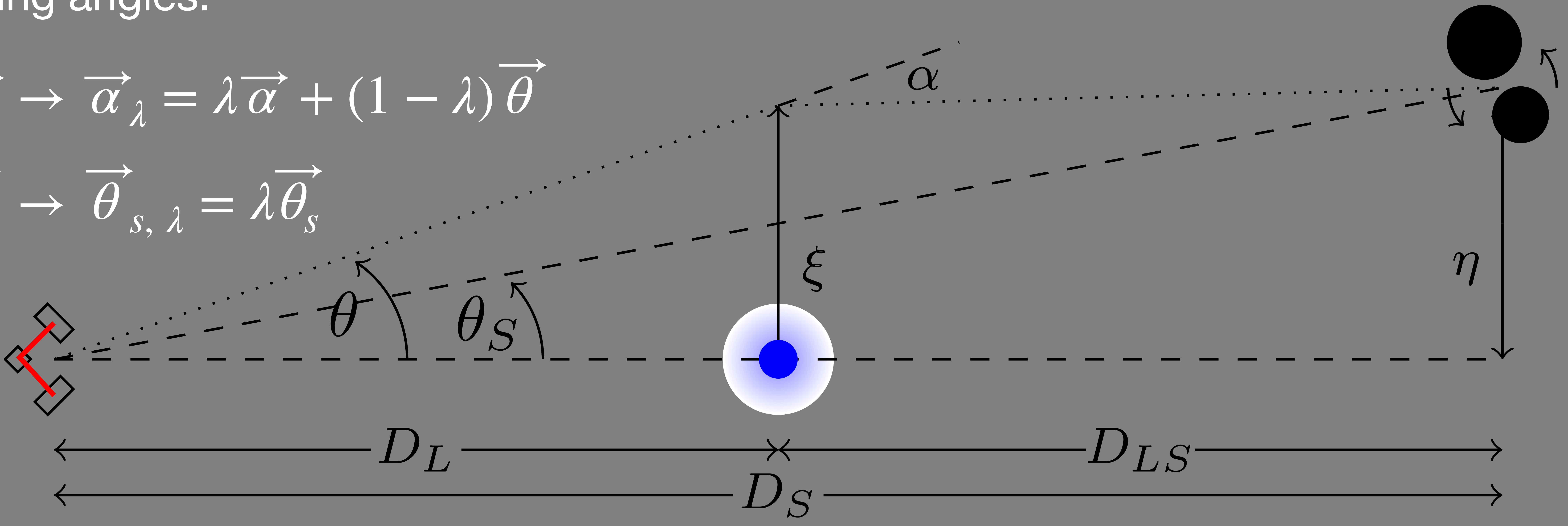
E. E. Falco, M. V. Gorenstein, and I. I. Shapiro, ApJ 289, L1 (1985)

- Scalings of lens mass:
  - $\kappa \rightarrow \kappa_\lambda = \lambda\kappa + (1 - \lambda)$

$\kappa = \Sigma / \Sigma_{cr}$

Σ - surface mass density

- Scaling angles:
  - $\vec{\alpha} \rightarrow \vec{\alpha}_\lambda = \lambda\vec{\alpha} + (1 - \lambda)\vec{\theta}$
  - $\vec{\theta}_s \rightarrow \vec{\theta}_{s,\lambda} = \lambda\vec{\theta}_s$



MSD

Why a problem?

- Observables are preserved!
- Problems: e.g. biased estimations of mass lens
- Biased estimation of cosmological parameter, e.g. H_0

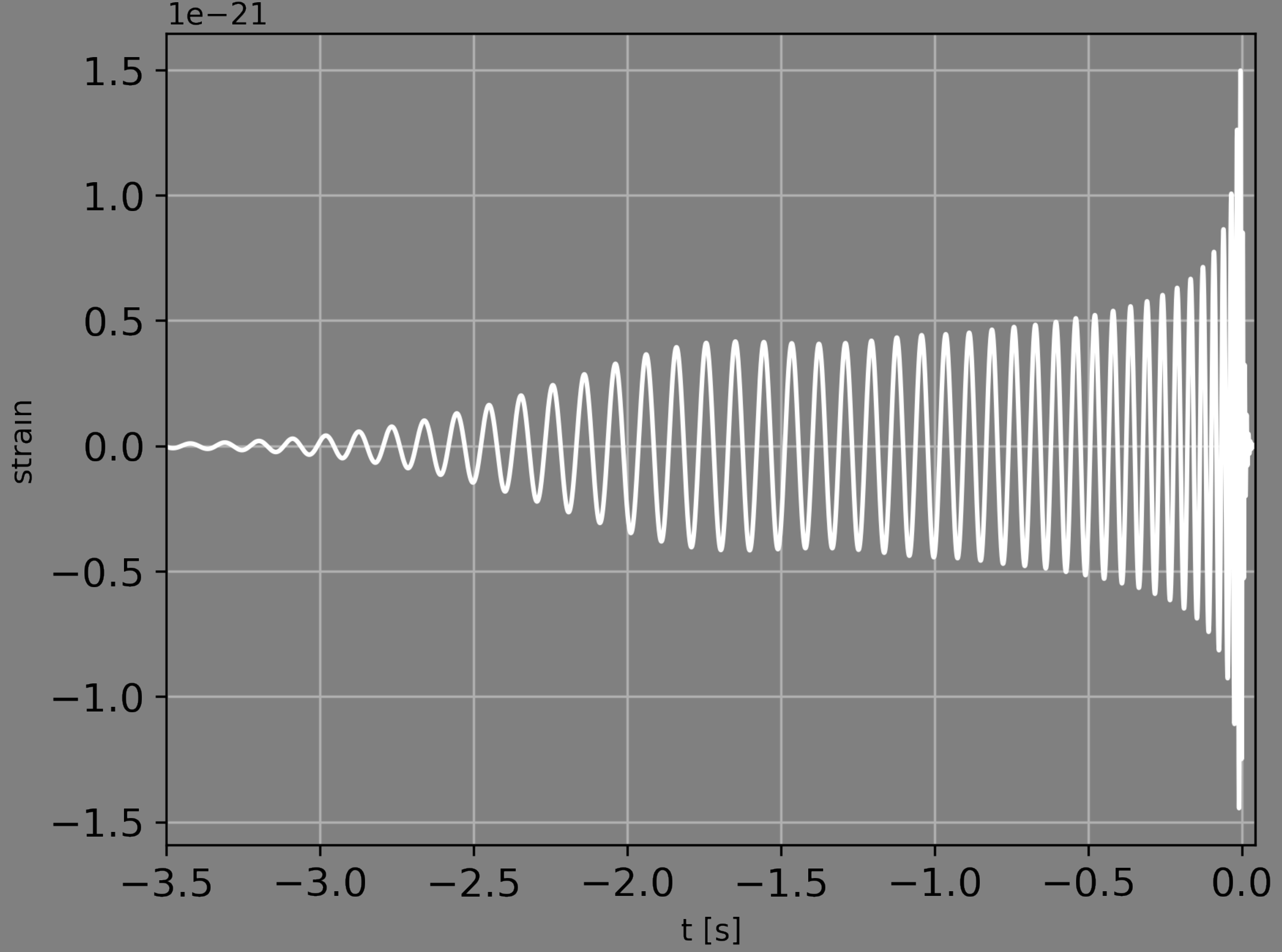
Can we solve it?

- EM geometrical optics regime: multiple images; independent mass estimation of the lens (e.g. dynamics)
- EM wave optics regime: multiple lenses
- **In GW lensing: 1 image and 1 lens can break MSD!**

Gravitational Waves Lensing

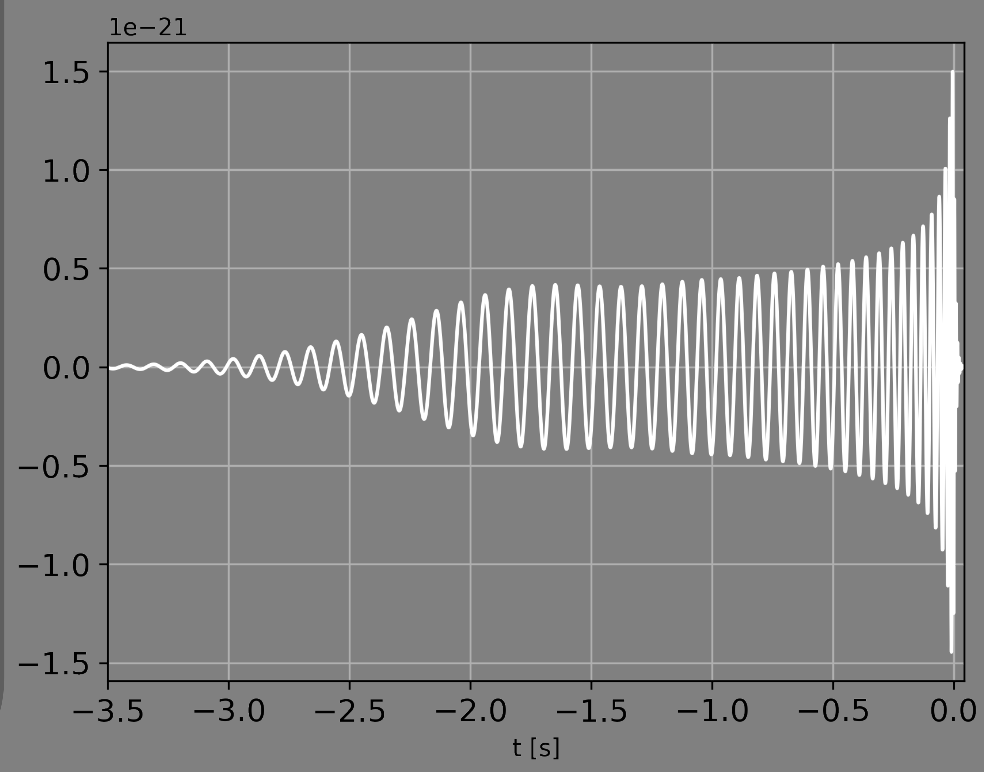
GL of GW

$$h(t)$$

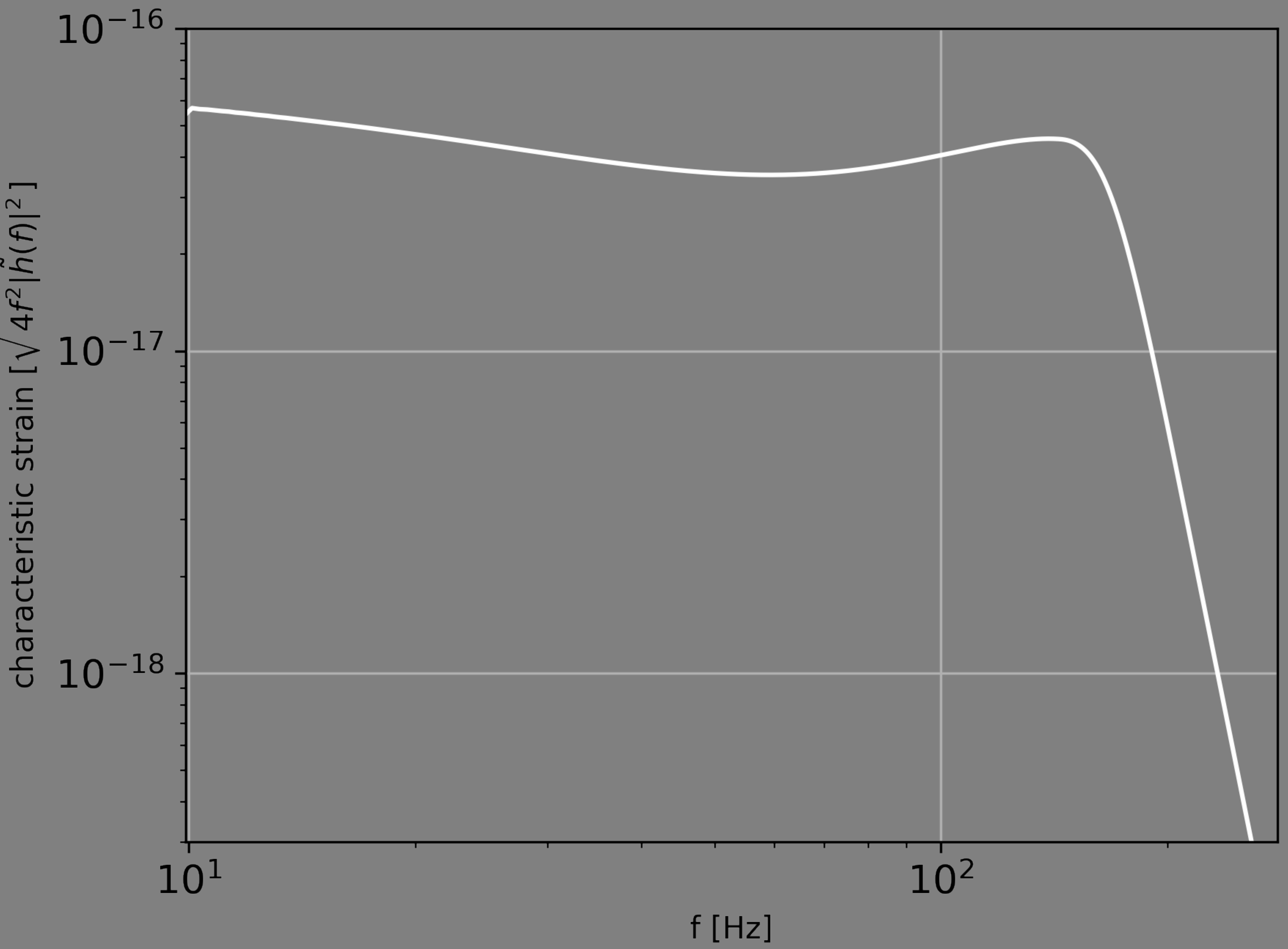


GL of GW

$$\int_{-\infty}^{\infty} h(t) \cdot e^{-i2\pi ft}dt = \tilde{h}(f)$$

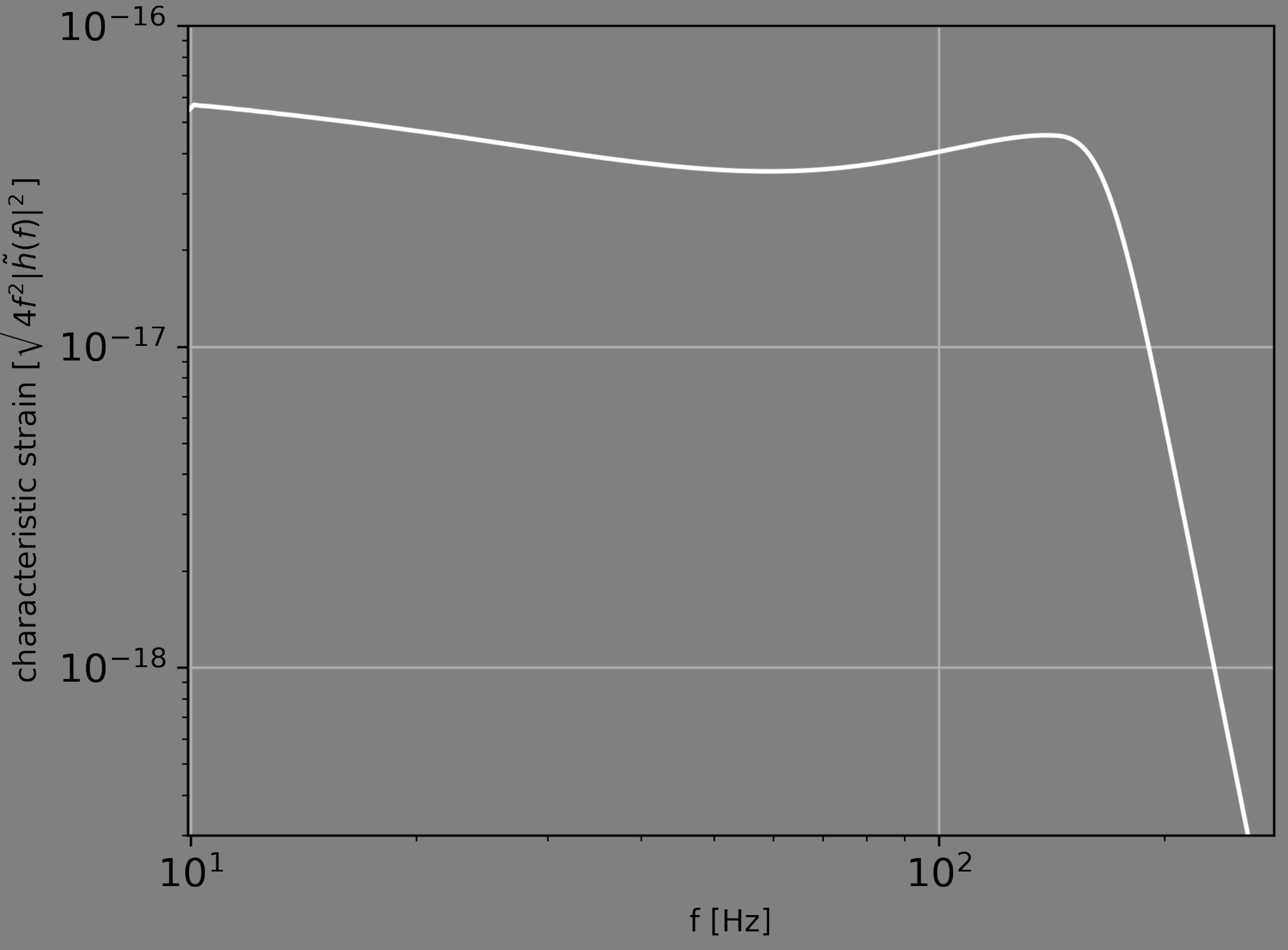
$\int$ 

$\cdot e^{-i2\pi ft}dt =$



GL of GW

$$\tilde{h}(f) \cdot F(\theta_s, f) = \tilde{h}_L(f)$$



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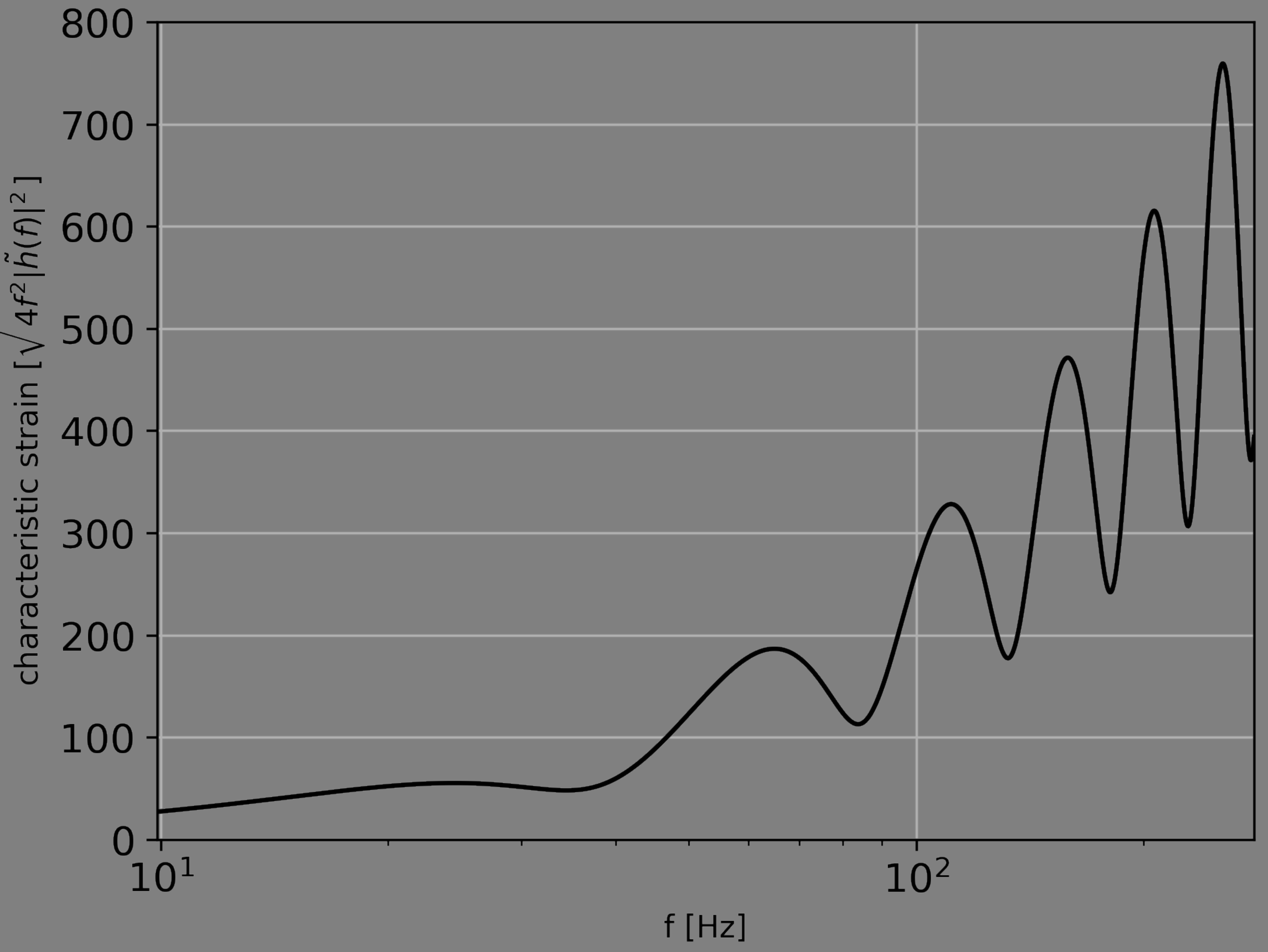

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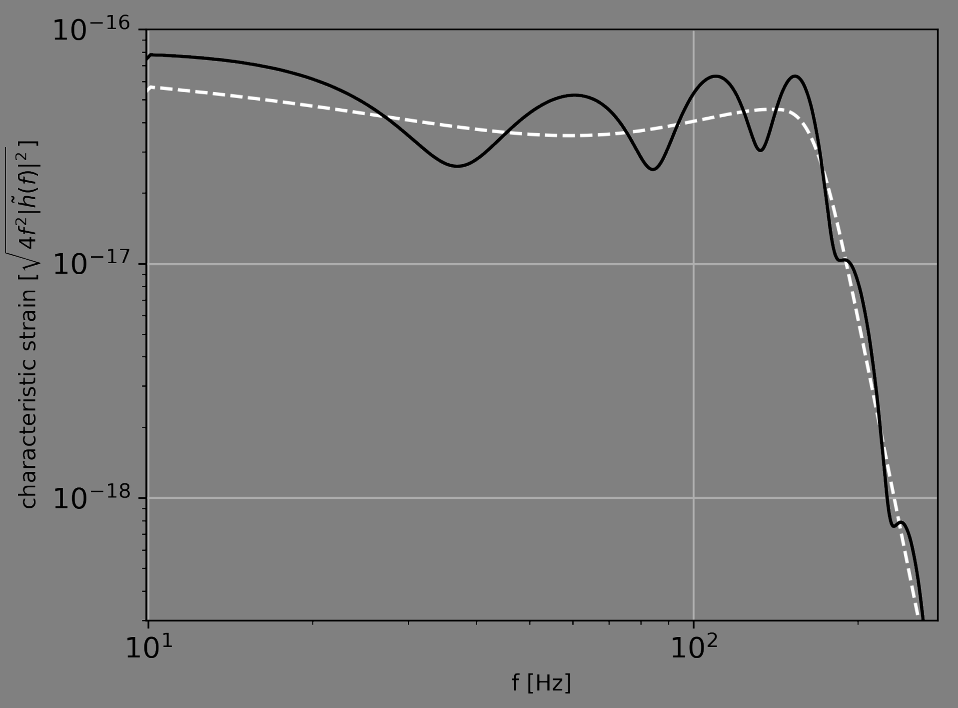
GL of GW

$$\tilde{h}(f) \cdot F(\theta_s, f) = \tilde{h}_L(f)$$

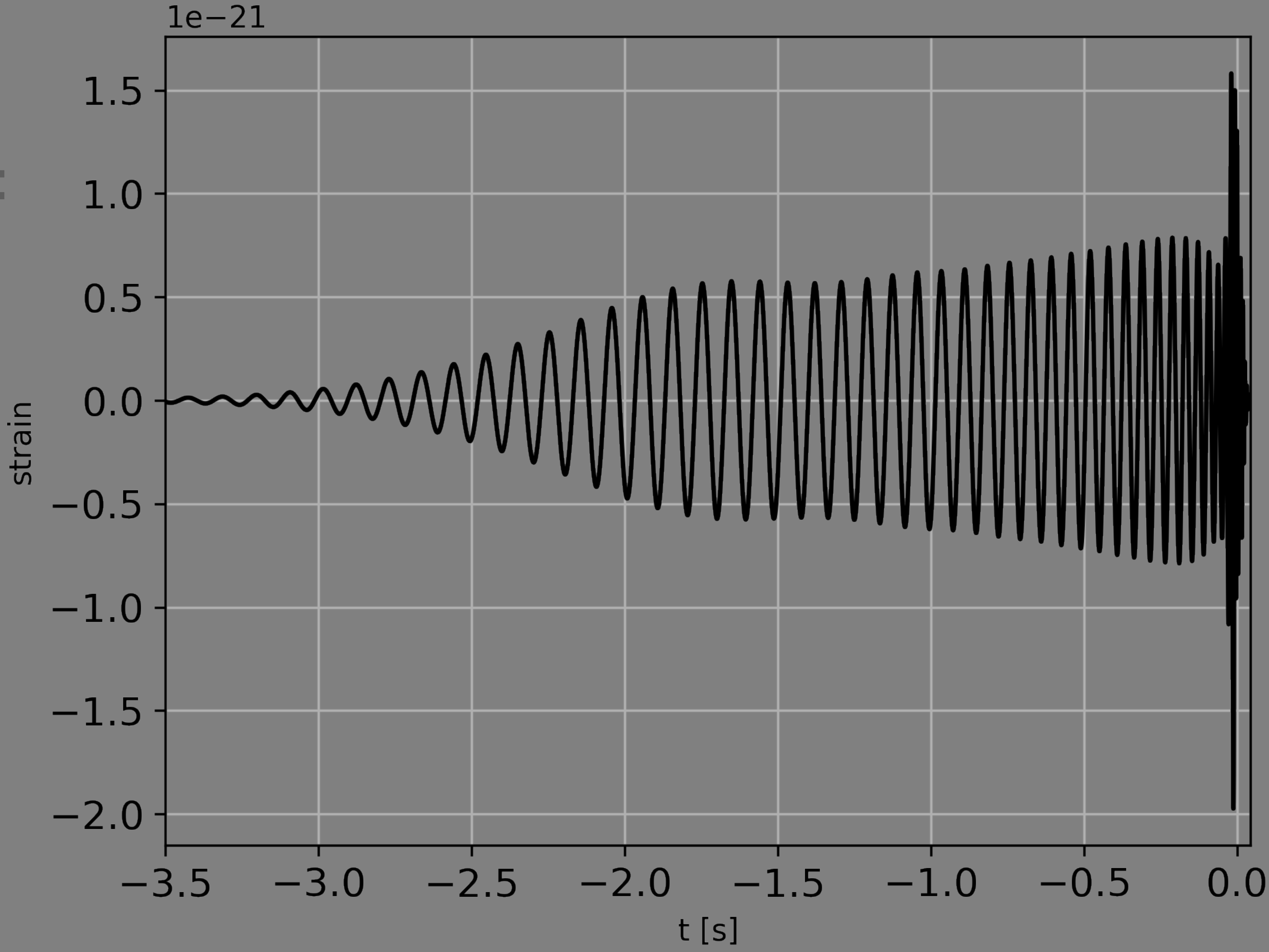
GL of GW

$$\int_{-\infty}^{\infty} \tilde{h}_L(f) \cdot e^{i2\pi f t} df = h_L(t)$$

$\int$

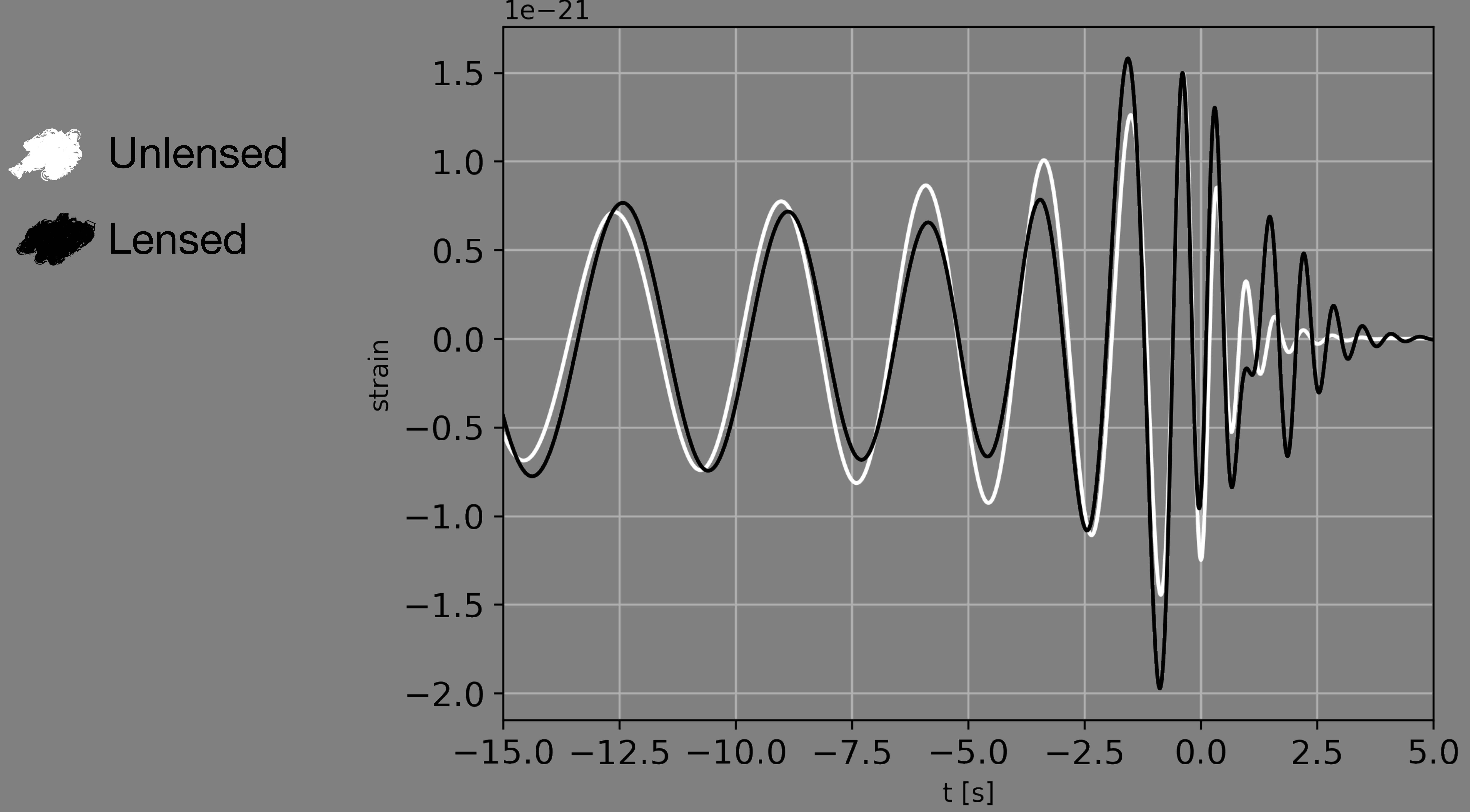


$\cdot\, e^{i2\pi f t} df$ $=$



GL of GW

$h_L(t)$ vs $h(t)$



Gravitational Lensing of Grav. Waves

NB: spherical symmetry!

- $\tilde{h}(f) \cdot F(f, \theta_s) = \tilde{h}_L(f)$
- $F(w, y) = -iwe^{iwy^2/2} \int_0^\infty \mathrm{d}x\, x\, J_0(wxy) \exp\left\{ iw \left[\frac{1}{2} x^2 - \Psi(x) \right] \right\} \rightarrow F_\lambda$

T. T. Nakamura and S. Deguchi, Progress of TheoreticalPhysics Supplement133, 137 (1999).

- Where:
  - $w = \frac{1 + z_L}{c} \frac{D_S D_L \theta_E^2}{D_{LS}} 2\pi f$
  - $x = |\vec{x}| = |\vec{\theta} / \vec{\theta}_E|$
  - $y = |\vec{y}| = |\vec{\theta}_s / \vec{\theta}_E| \rightarrow y_\lambda$

- J_0 - Bessel function of 0-th order
- Ψ - dimensionless effective lensing potential $\rightarrow \Psi_\lambda$

Lensed waveforms under mass-sheet transformation

Qualitative analysis

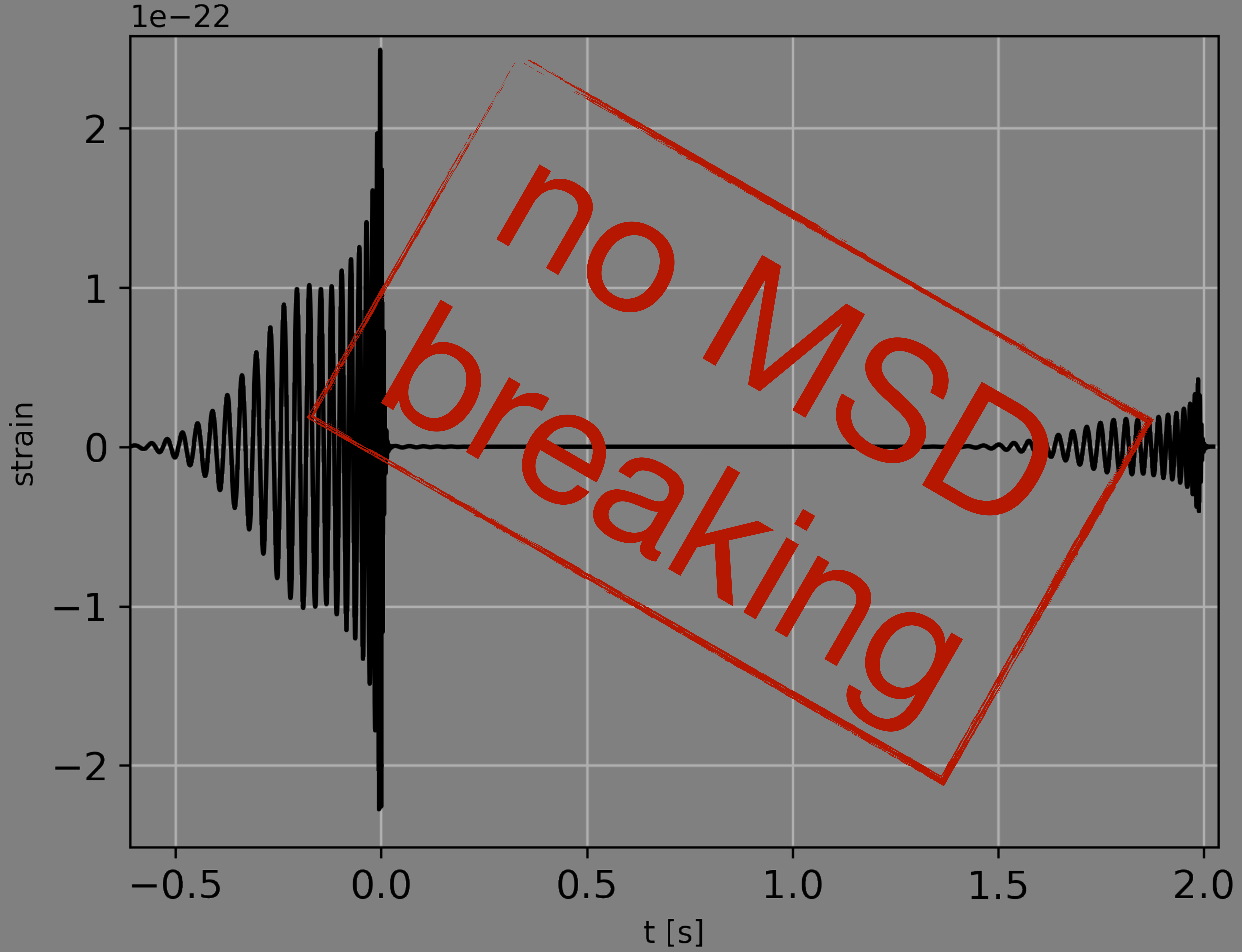
Lensed GWs

3 regimes

- Geometrical Optics
  - $f \cdot \Delta t \gg 1$
  - $M_L > 10^5[(1+z_L)f]^{-1}$

$M_S = 60\ M_\odot\ -\ z_S = 0.5$

$M_L = 10^4\ M_\odot\ -\ z_L = 0.1\ -\ y = 5$

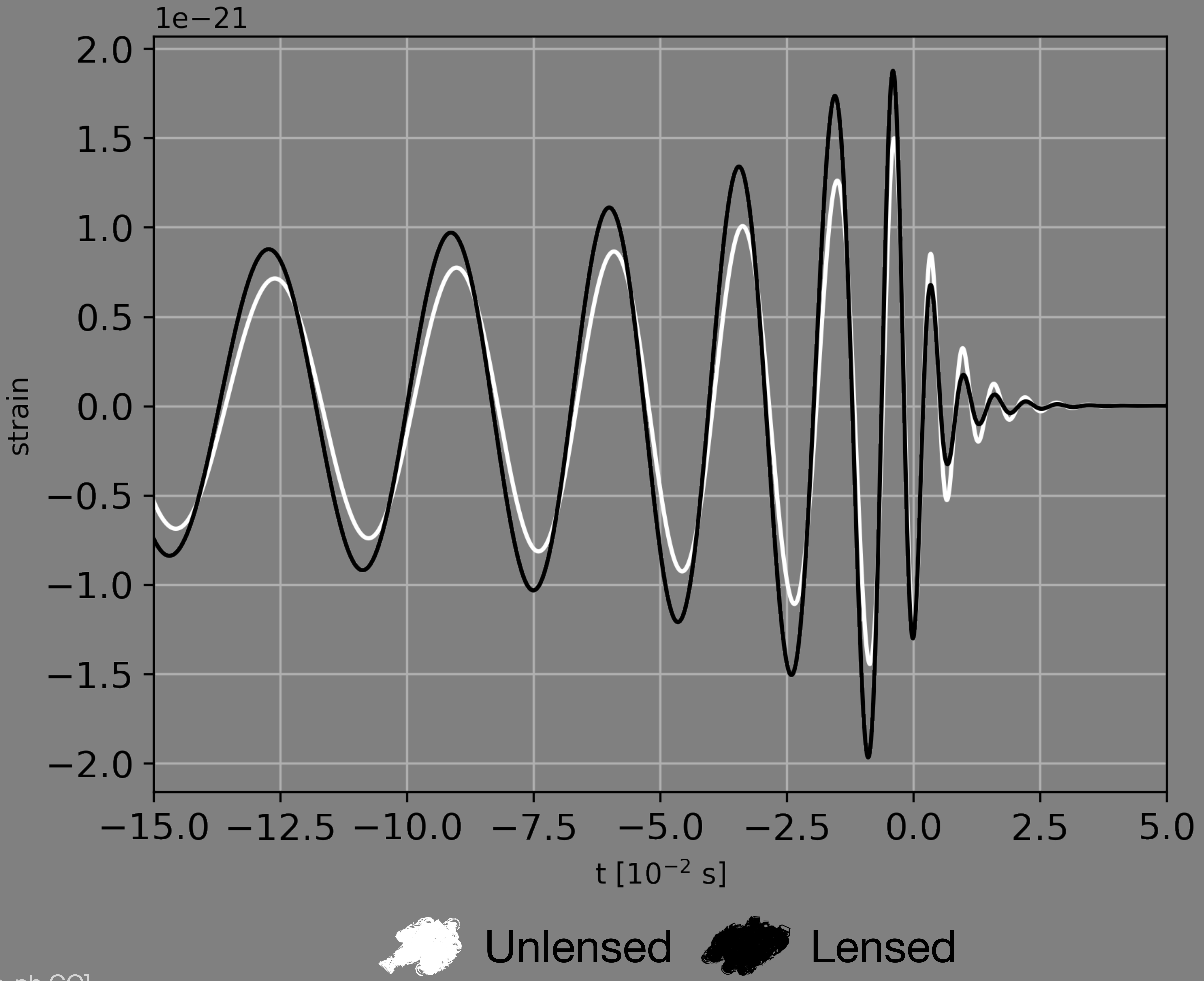


R.Takahashi,Astrophys.J.835,103(2017),arXiv:1606.00458 [astro-ph.CO].

Lensed GWs

3 regimes

- Wave Optics
  - $f \cdot \Delta t \lesssim 1$
  - $M_L \leq 10^5[(1+z_L)f]^{-1}$



$M_S = 100\ M_\odot\ -\ z_S = 0.1$

$M_L = 100\ M_\odot\ -\ z_L = 0.01$

R.Takahashi,Astrophys.J.835,103(2017),arXiv:1606.00458 [astro-ph.CO].

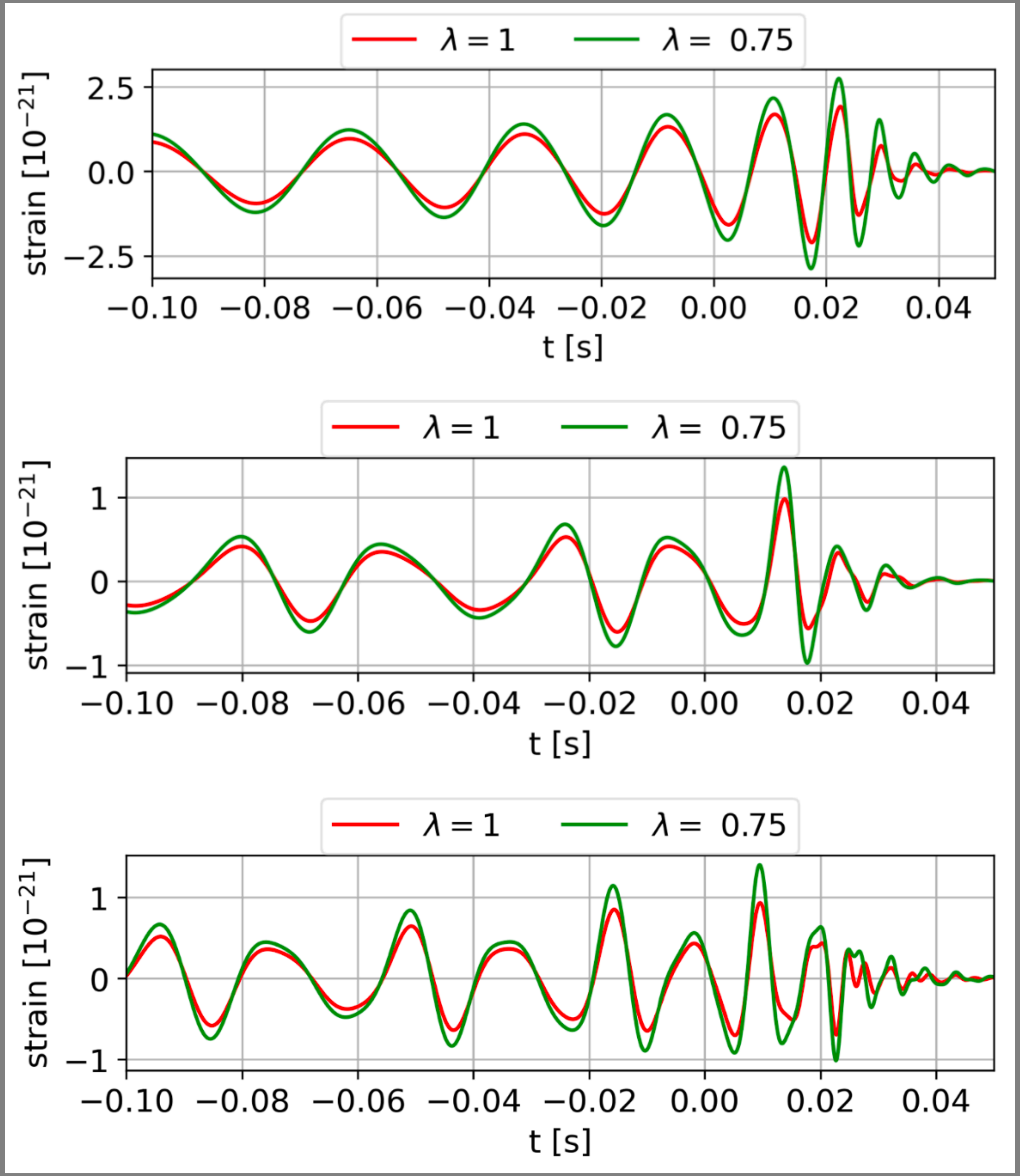
Lensed GWs

Wave optics

$$q = \frac{m_2}{m_1} = 1$$

$$q = 0.1$$

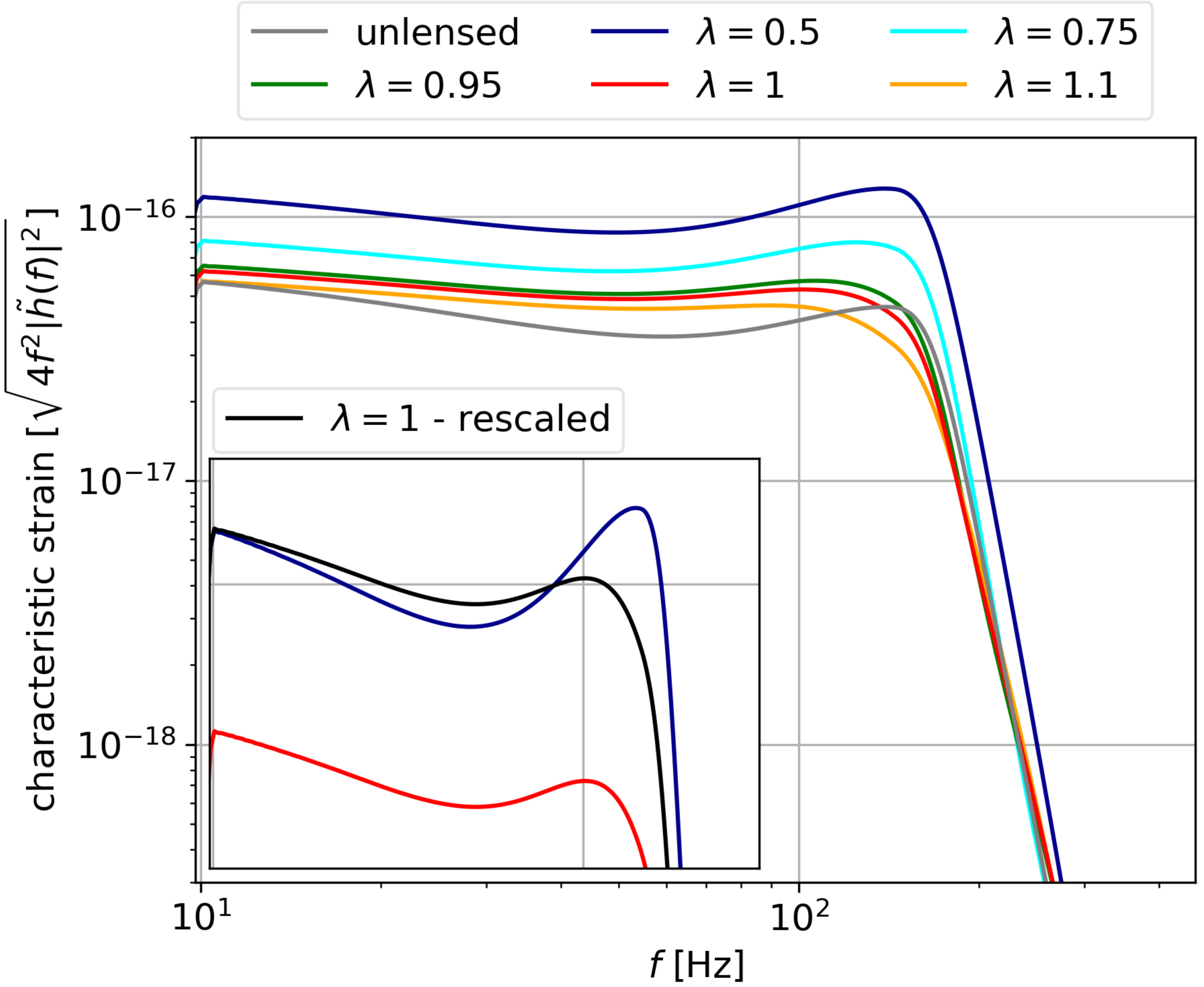
$$q = 0.1 \ \& \ s_{1,2;z} = \{0.7, 0.2\}$$



Lensed GWs

Wave optics

$$q = 1$$



Lensed GWs

3 regimes

Interference regime: $f \cdot \Delta t \approx 1$

$M_L = 500\ M_\odot$ & $y = 1$ & $z_L = 0.01$

$M_S = 100\ M_\odot$ & $q = 1$ & $z_S = 0.1$

Lensed GWs

3 regimes

Interference regime: $f \cdot \Delta t \approx 1$

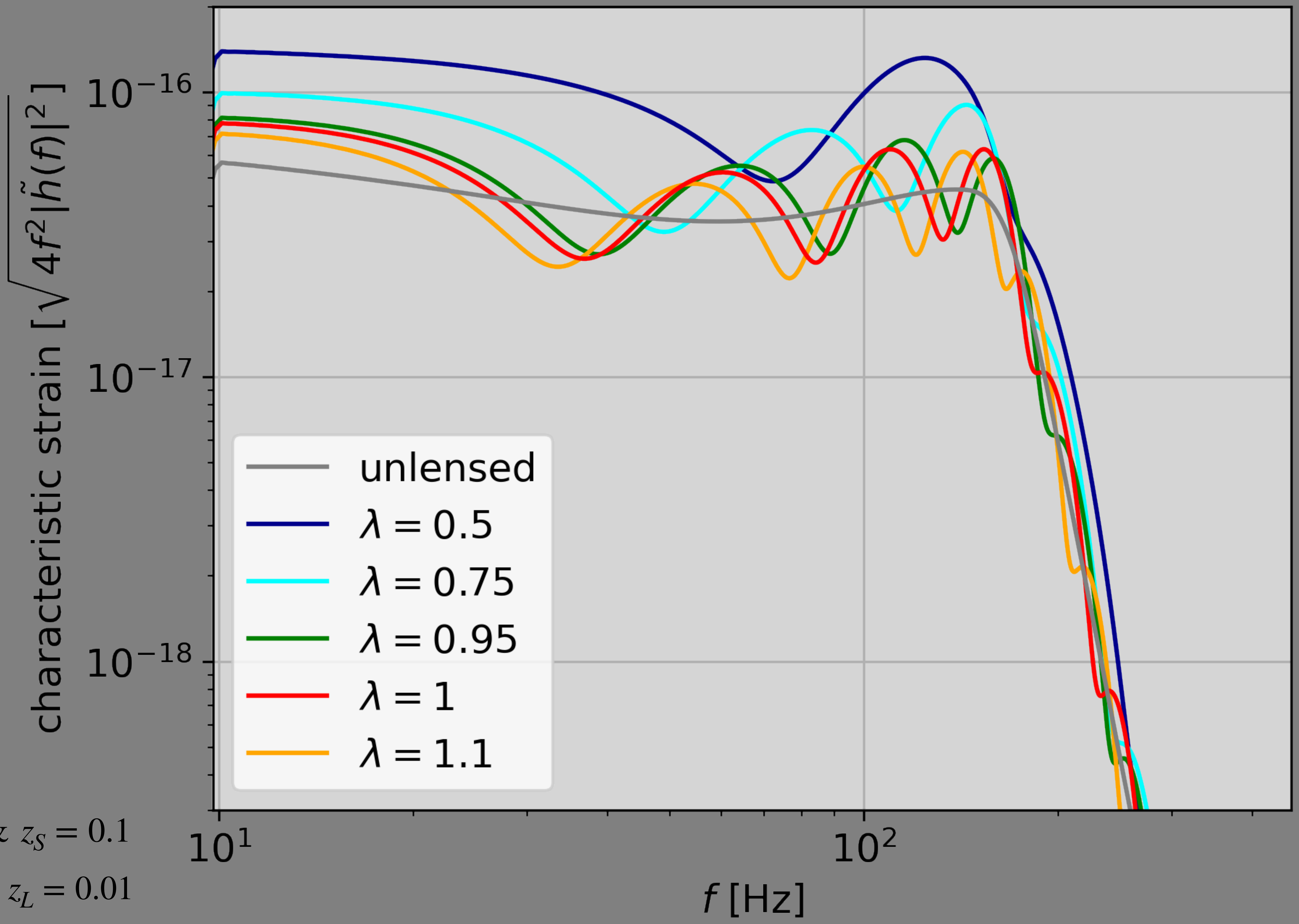
$M_L = 500\ M_\odot$ & $y = 1$ & $z_L = 0.01$

$M_S = 100\ M_\odot$ & $q = 1$ & $z_S = 0.5$

Lensed GWs

Interference regime

- $f \cdot \Delta t \approx 1$



$M_S = 100\ M_\odot$ & $q = 1$ & $z_S = 0.1$

$M_L = 500\ M_\odot$ & $y = 1$ & $z_L = 0.01$

S/N - template matching

Quantitative analysis

S/N

data from
https://www.ligo.caltech.edu/page/ligo-data

Signal-to-Noise ratio

$$\rho = \frac{(s\,|\,h_T)}{\sqrt{(h_T\,|\,h_T)}} \approx \frac{(h\,|\,h_T)}{\sqrt{(h_T\,|\,h_T)}}$$

- $s(t) = h(t) + n(t)$
- Inner product:

$$(a\,|\,b) = 4\,\mathrm{Re}\left[\int_0^\infty \frac{\tilde{a}(f)\cdot\tilde{b}^*(f)}{S_n(f)}df\right]$$

  - $S_n(f)$ - (single-sided) power spectral density (L1-O3-LIGO)

Confidence region: $$\Delta\chi^2 \approx 2\rho_{opt}^2\left[1 - \frac{\rho}{\rho_{opt}}\right]$$

$$3\sigma \rightarrow \Delta\chi^2 \approx 11.8$$

M. Maggiore,Gravitational Waves: Volume 1: Theory and Experiments,Gravitational Waves (OUP Oxford, 2008)

S/N

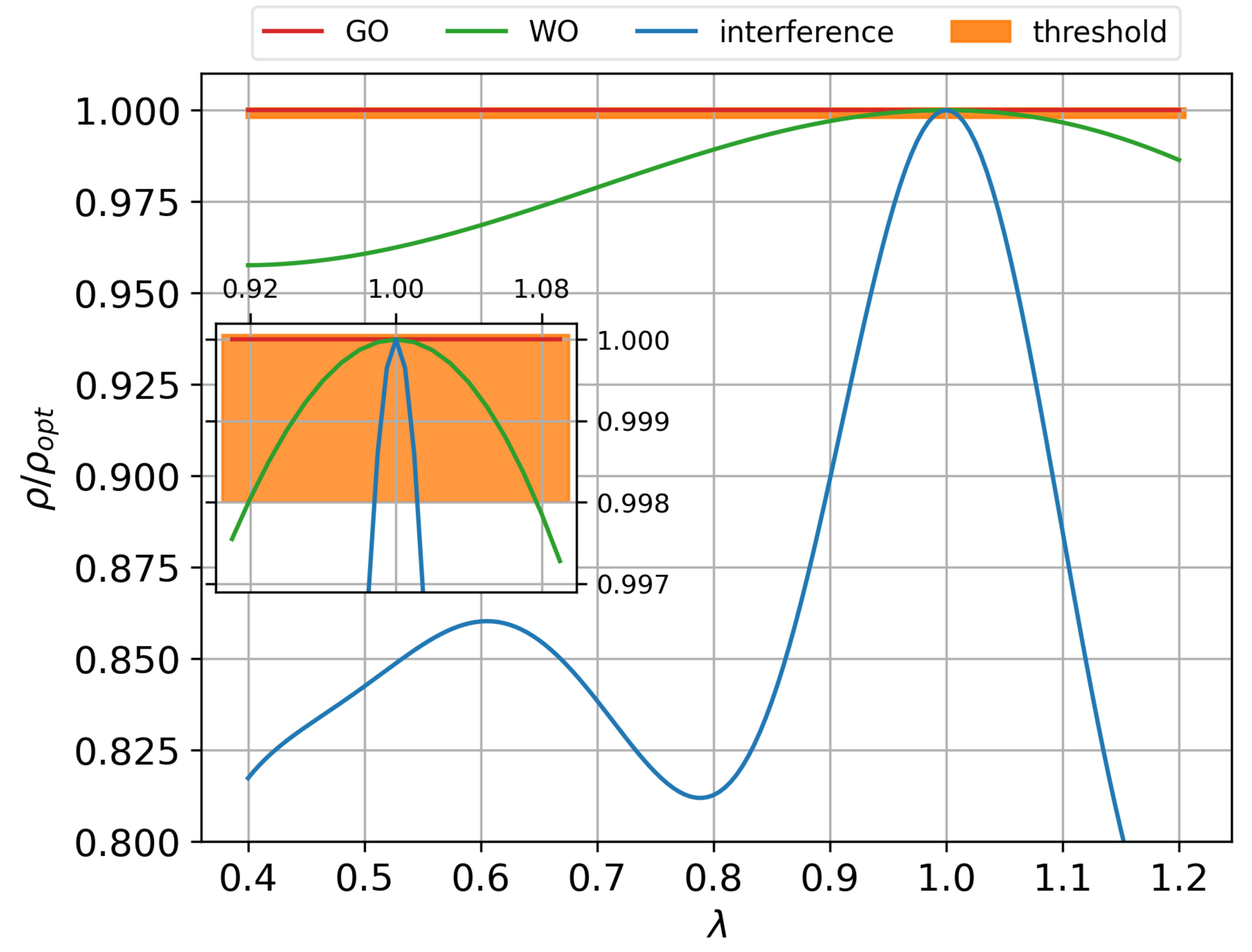
- $M_S = 100\ M_\odot$
- $z_S = 0.1$
- $z_L = 0.01$

$3\sigma \rightarrow \Delta\chi^2 \approx 0.998$

GO: $M_L = 500 M_\odot$, $y = 10$

Int.: $M_L = 500 M_\odot$, $y = 1$

WO: $M_L = 100 M_\odot$, $y = 1$



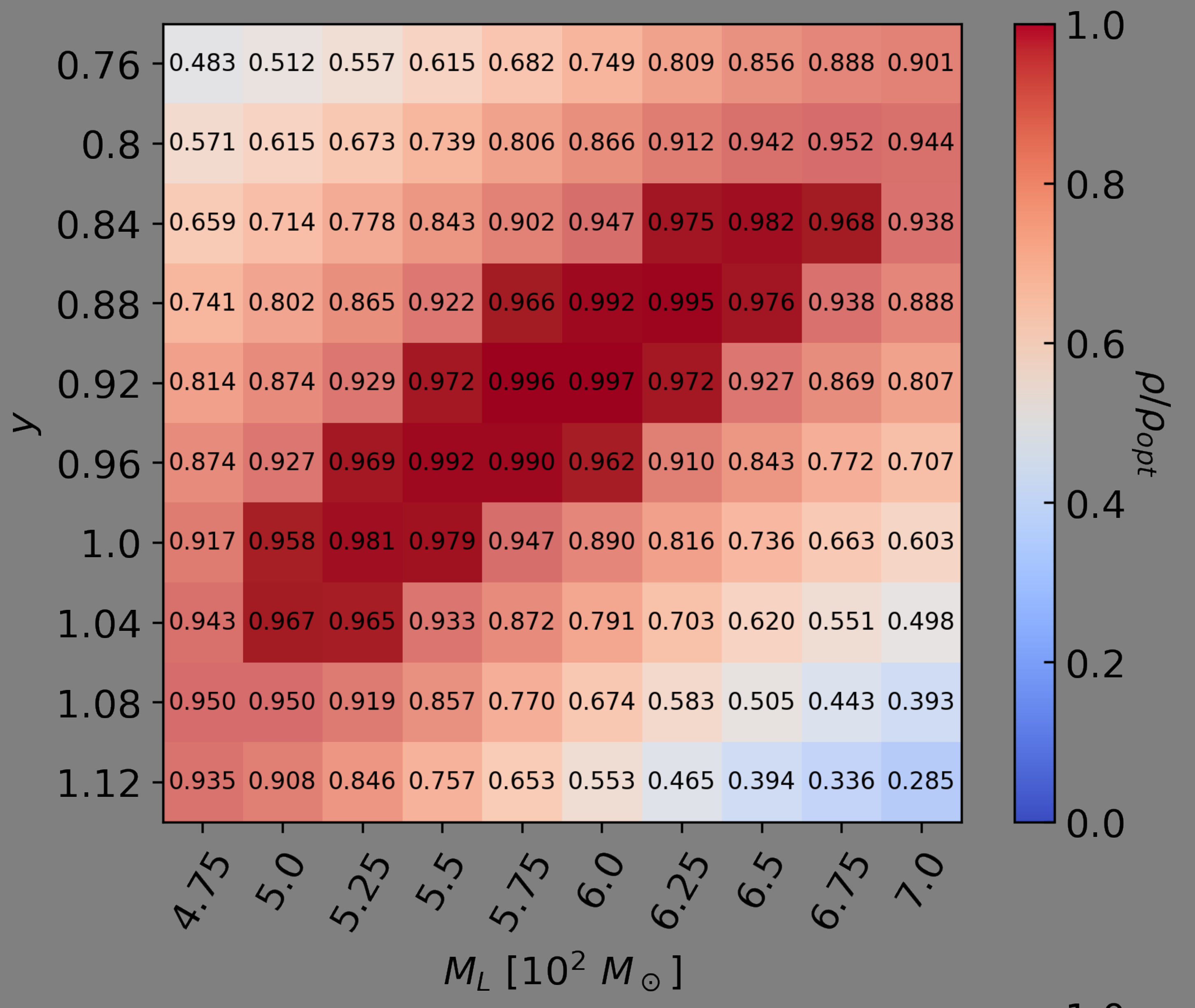
S/N
$\lambda_{min} = 0.93$
$\lambda = 1$
$\lambda_{max} = 1.03$
Interference regime
Signal
$M_L = 500$
$y = 1$
$z_S = 0.5$
$\rho_{opt} \approx 11$
Signal
$M_L = 500$
$y = 1$
$z_S = 0.1$
$\rho_{opt} \approx 55$
y
M_L [10^2 $M_\odot$]
ρ/ρ_{opt}
$\lambda_{min} = 0.99$
$\lambda = 1$
$\lambda_{max} = 1.01$

S/N

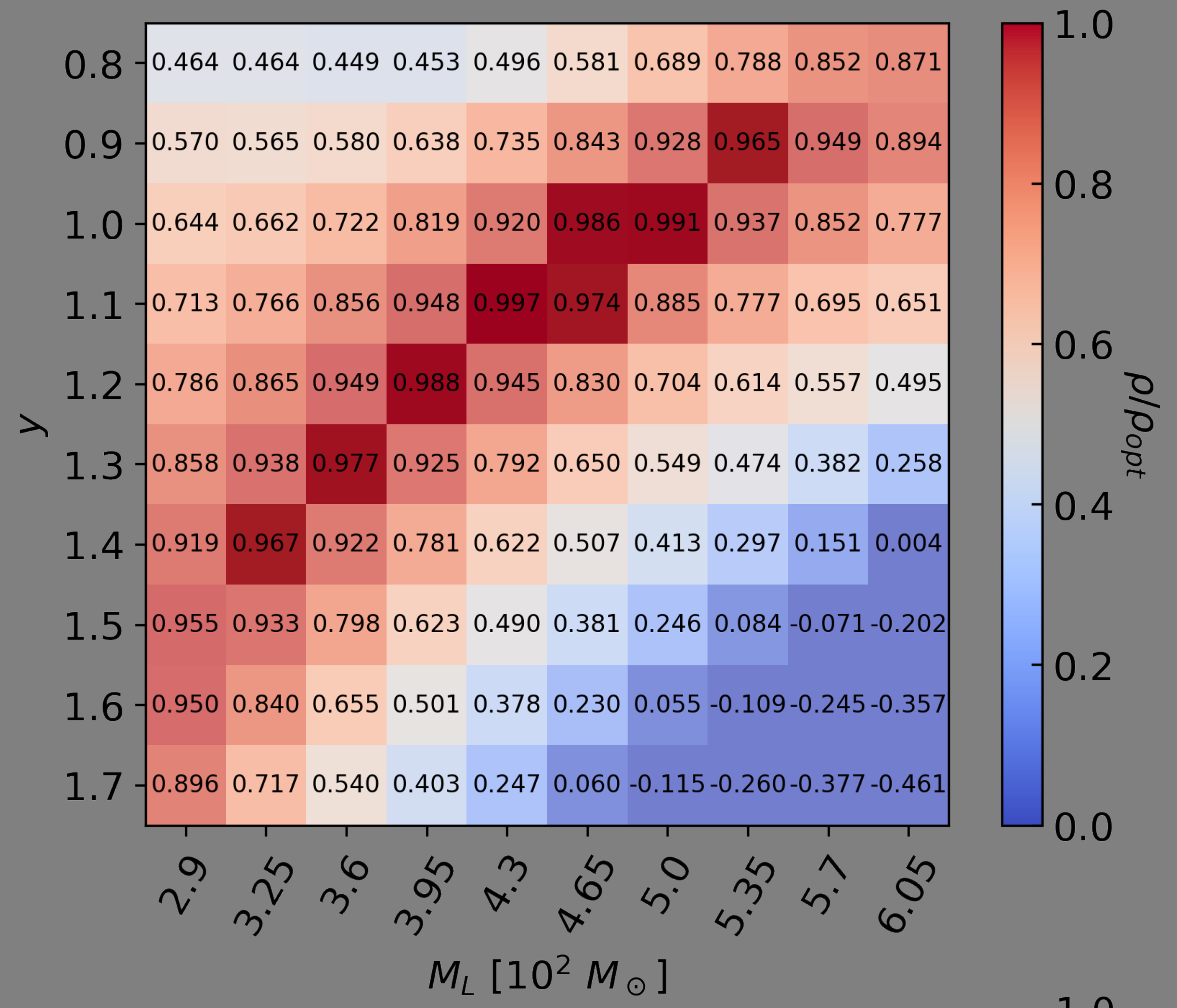
Interference regime

$\rho_{opt} \approx 11$

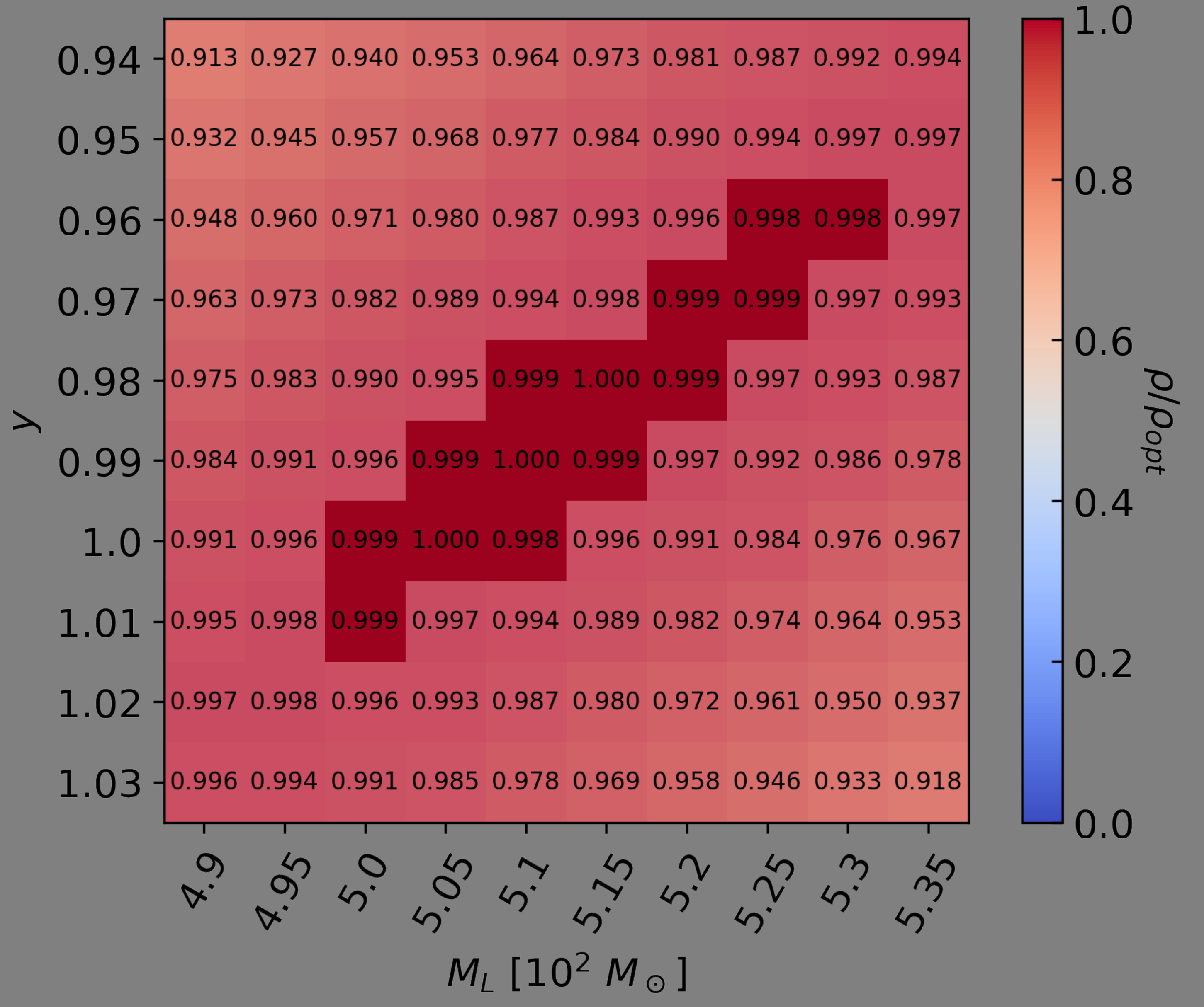
$\lambda_{min} = 0.93$



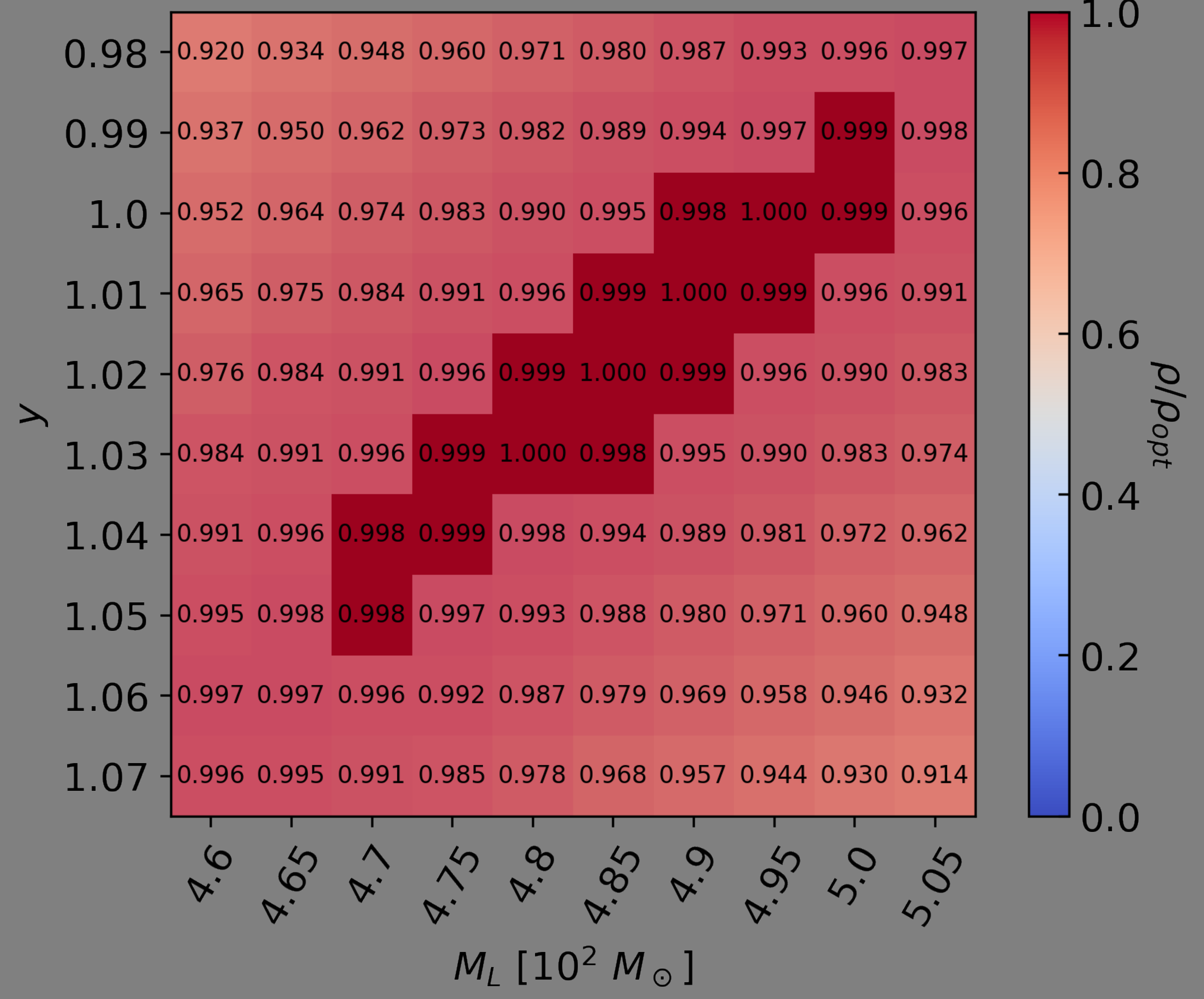
$\lambda_{max} = 1.03$



$\rho_{opt} \approx 55$



$\lambda_{min} = 0.99$



$\lambda_{max} = 1.01$

$\Delta y < 40\,\%$
$\Delta M_L \approx 35\,\%$

$>$

$\Delta M_L \approx 12 - 20\,\%$

$>$

$\Delta y \approx 5\,\%$
$\Delta M_L \approx 6\,\%$

P. Schneider and D. Sluse, Astron. Astrophys.559, A37(2013), arXiv:1306.0901 [astro-ph.CO].

Conclusions

Conclusions

1. We analysed how MSD behave in GW lensing
2. In the GO regime it can not be broken
3. In WO can be broken in some cases
4. In interference regime is broken
5. How well it is broken depends on the strength of the signal and sensitivity of detectors. Nowadays we might have up to $\Delta y \approx 5\,\%$ and $\Delta M \approx 6\,\%$

Contacts

- paolo.cremonese@usz.edu.pl
- paolocremonese.com