IberiCOS 2021

Breaking the Mass Sheet **Degeneracy with Gravitational Waves** Interference in Lensing events

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01/04/2021 - Paolo Cremonese





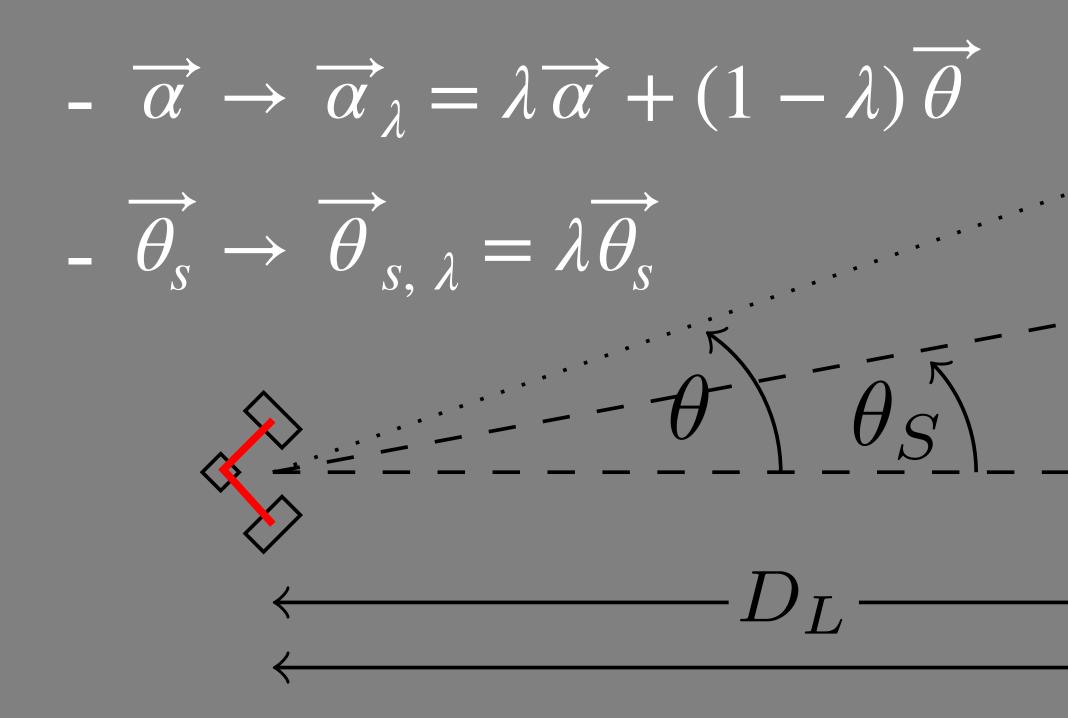


E. E. Falco, M. V. Gorenstein, and I. I. Shapiro, ApJ 289, L1 (1985)

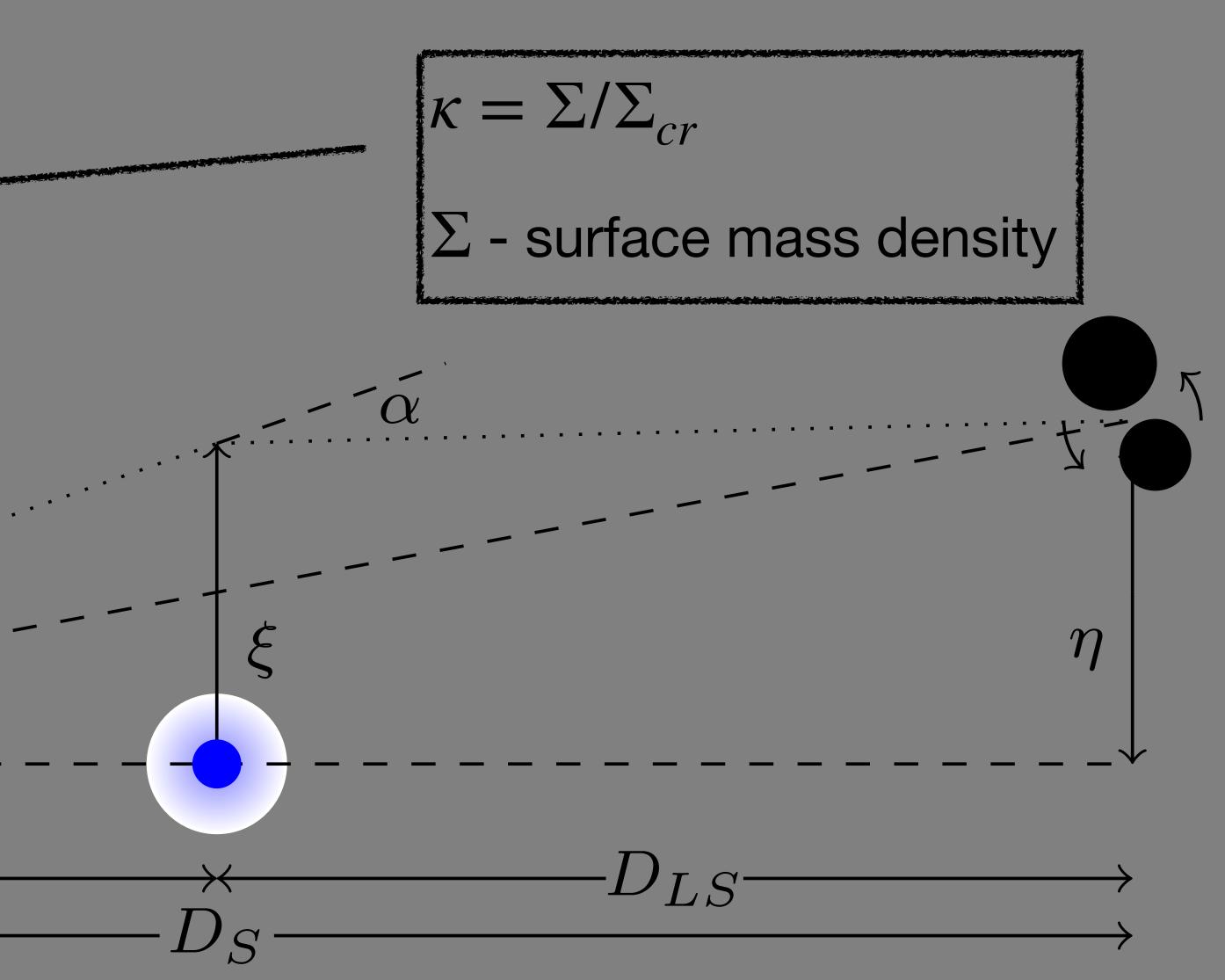
• Scalings of lens mass:

$$- \kappa \to \kappa_{\lambda} = \lambda \kappa + (1 - \lambda)$$

• Scaling angles:



Mass Sheet Degeneracy



Why a problem?

- Observables are preserved!
- Problems: e.g. biased estimations of mass lens
- Biased estimation of cosmological parameter, e.g. H_0

MSD

Can we solve it?

- EM geometrical optics regime: multiple images; independent mass estimation of the lens (e.g. dynamics)
- EM wave optics regime: multiple lenses
- In GW lensing: 1 image and 1 lens can break MSD!





























Gravitational Waves Lensing



Gravitational Lensing of Grav. Waves

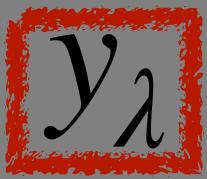
- $\tilde{h}(f) \cdot F(f, \theta_s) = \tilde{h}_L(f)$
- $F(w, y) = -iwe^{iwy^2/2} \int_{0}^{\infty} dx \, x J_0(w.$
- Where:
- $w = \frac{1 + z_L}{c} \frac{D_S D_L \theta_E^2}{D_L c} 2\pi f$ • $x = |\overrightarrow{x}| = |\overrightarrow{\theta}/\overrightarrow{\theta}_E|$ • $y = |\overrightarrow{y}| = |\overrightarrow{\theta_s}/\overrightarrow{\theta_E}| \longrightarrow v_2$

NB: spherical symmetry!

$$exy)\exp\left\{iw\left[\frac{1}{2}x^2-\Psi(x)\right]\right\}\longrightarrow F$$

T. T. Nakamura and S. Deguchi, Progress of TheoreticalPhysics Supplement133, 137 (1999).

- J_0 Bessel function of 0-th order
- Ψ dimensionless effective lensing potential







Lensed waveforms under mass-sheet transformation

Qualitative analysis



Lensed GWs 3 regimes

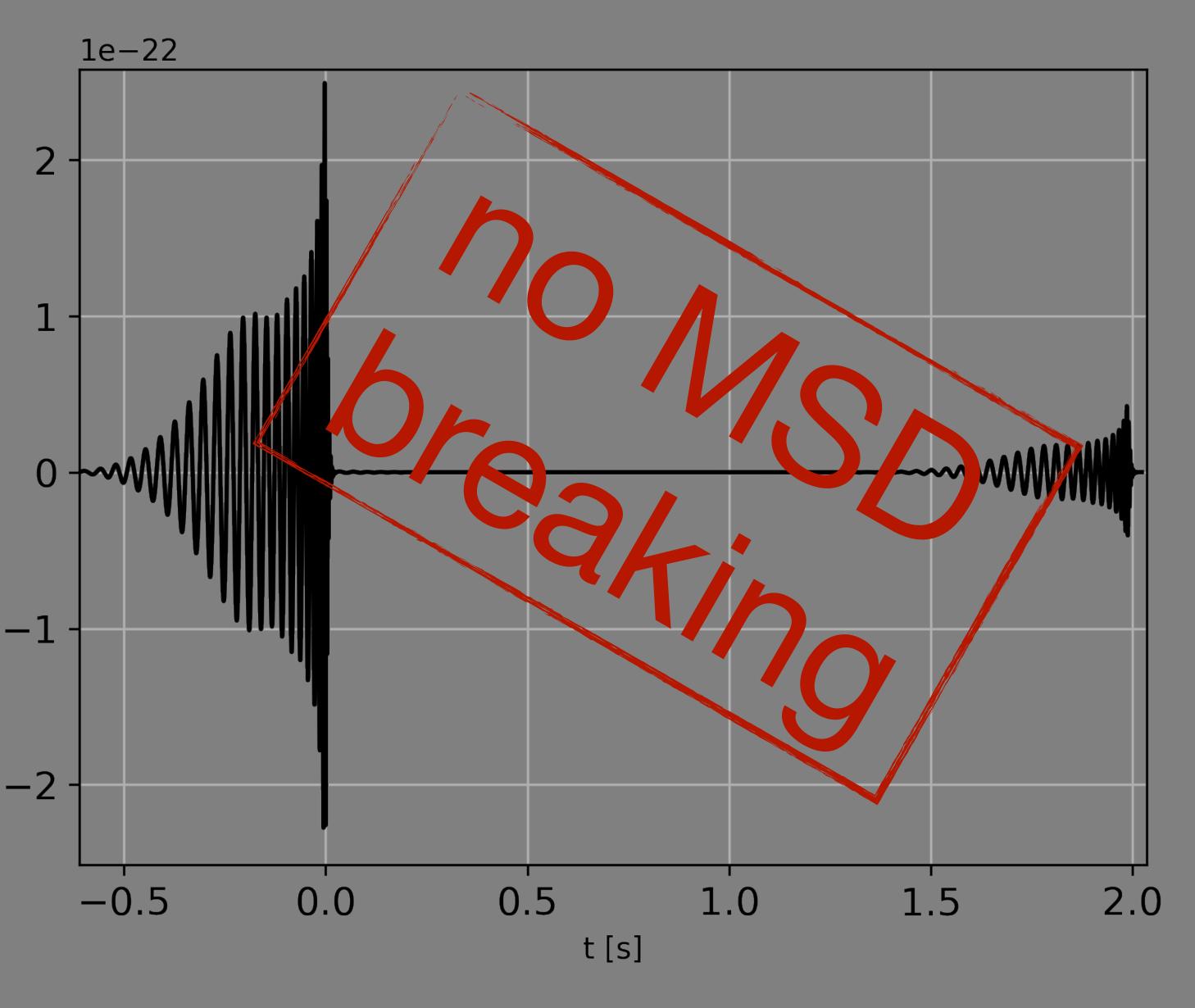
Geometrical Optics

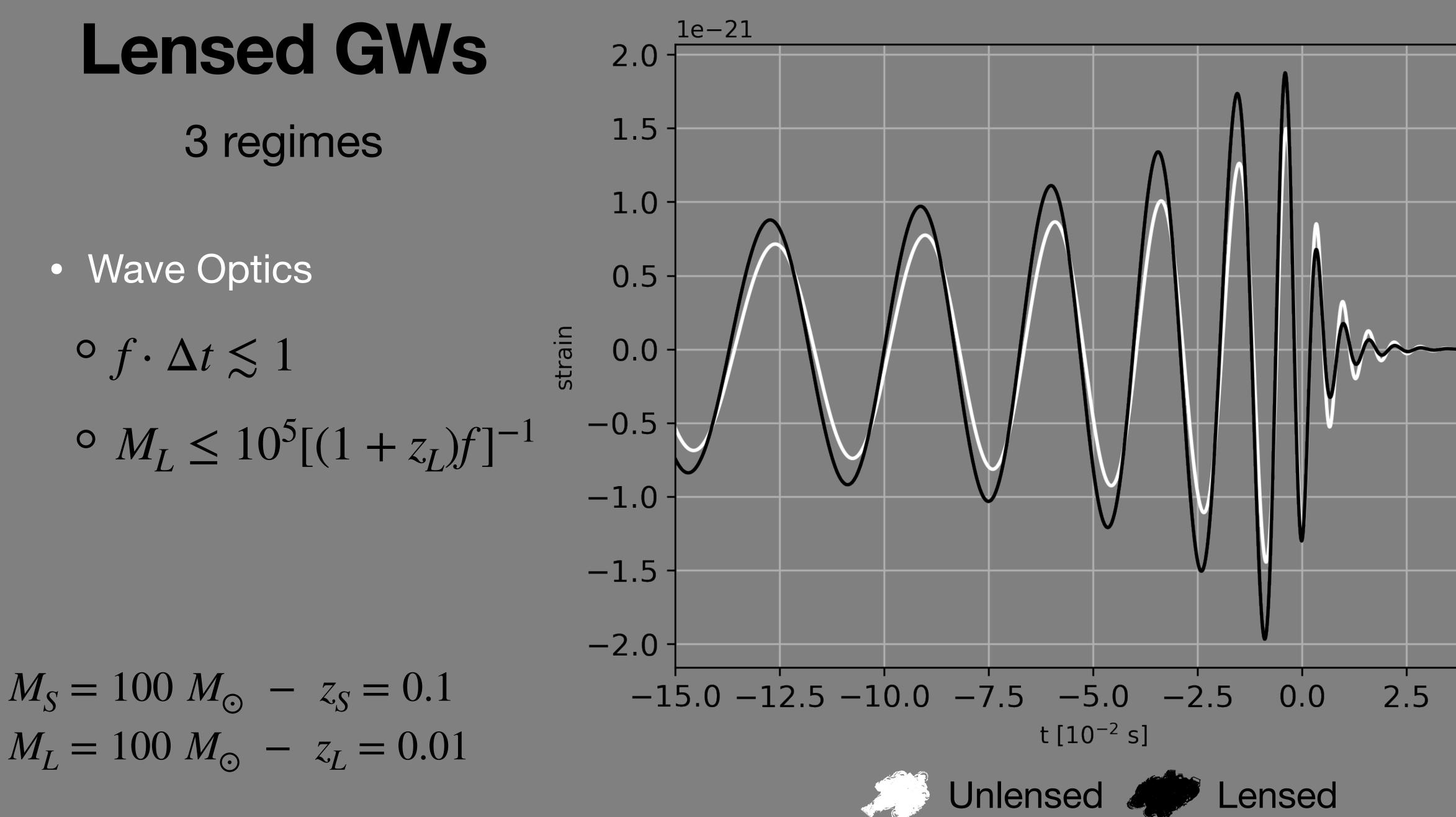
• $f \cdot \Delta t \gg 1$ • $M_L > 10^5 [(1 + z_L)f]^{-1}$

strain

$M_S = 60 \ M_{\odot} - z_S = 0.5$ $M_L = 10^4 \ M_{\odot} - z_L = 0.1 - y = 5$

R.Takahashi,Astrophys.J.835,103(2017),arXiv:1606.00458 [astro-ph.CO].

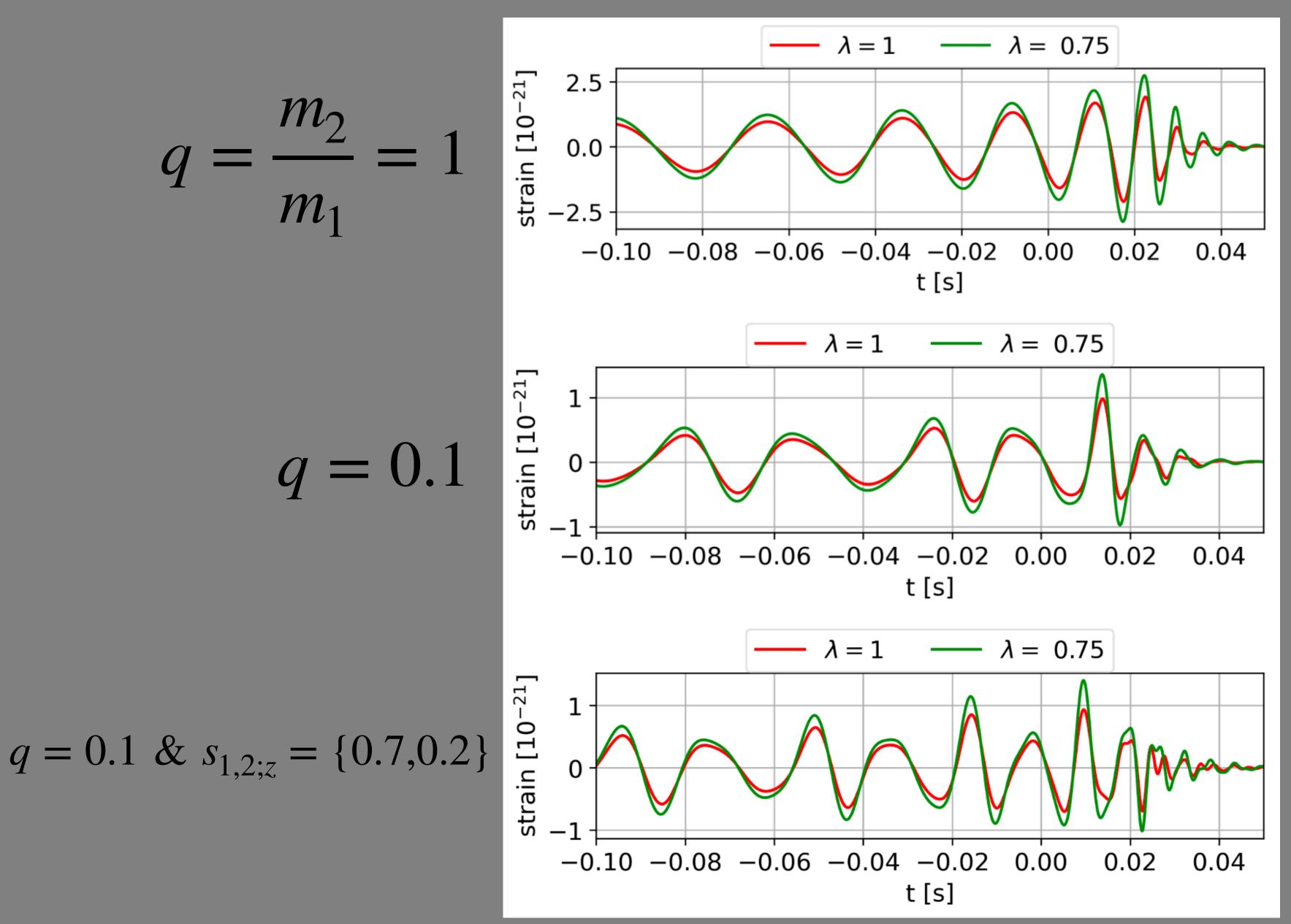




R.Takahashi,Astrophys.J.835,103(2017),arXiv:1606.00458 [astro-ph.CO].



Lensed GWs



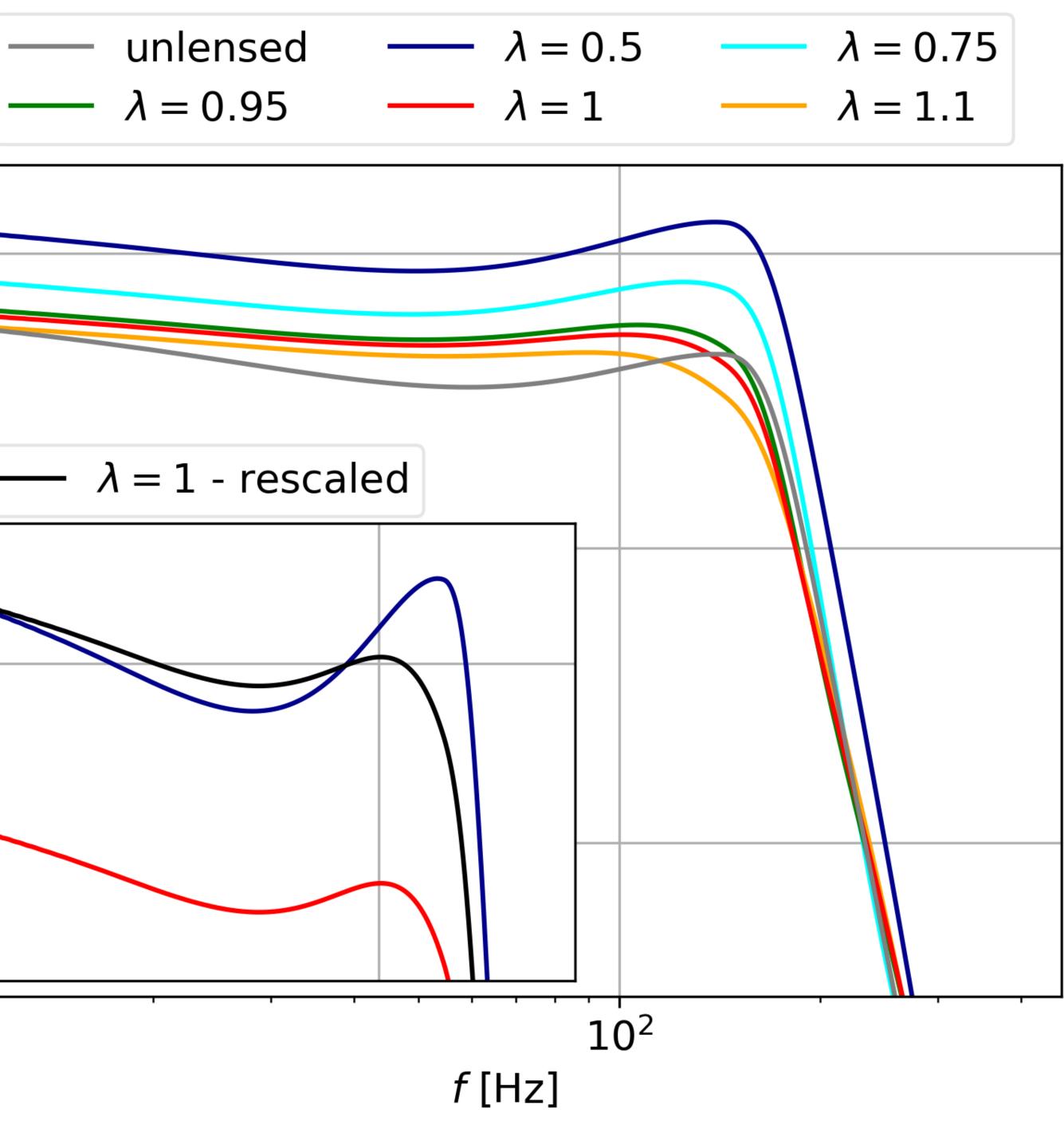
Wave optics

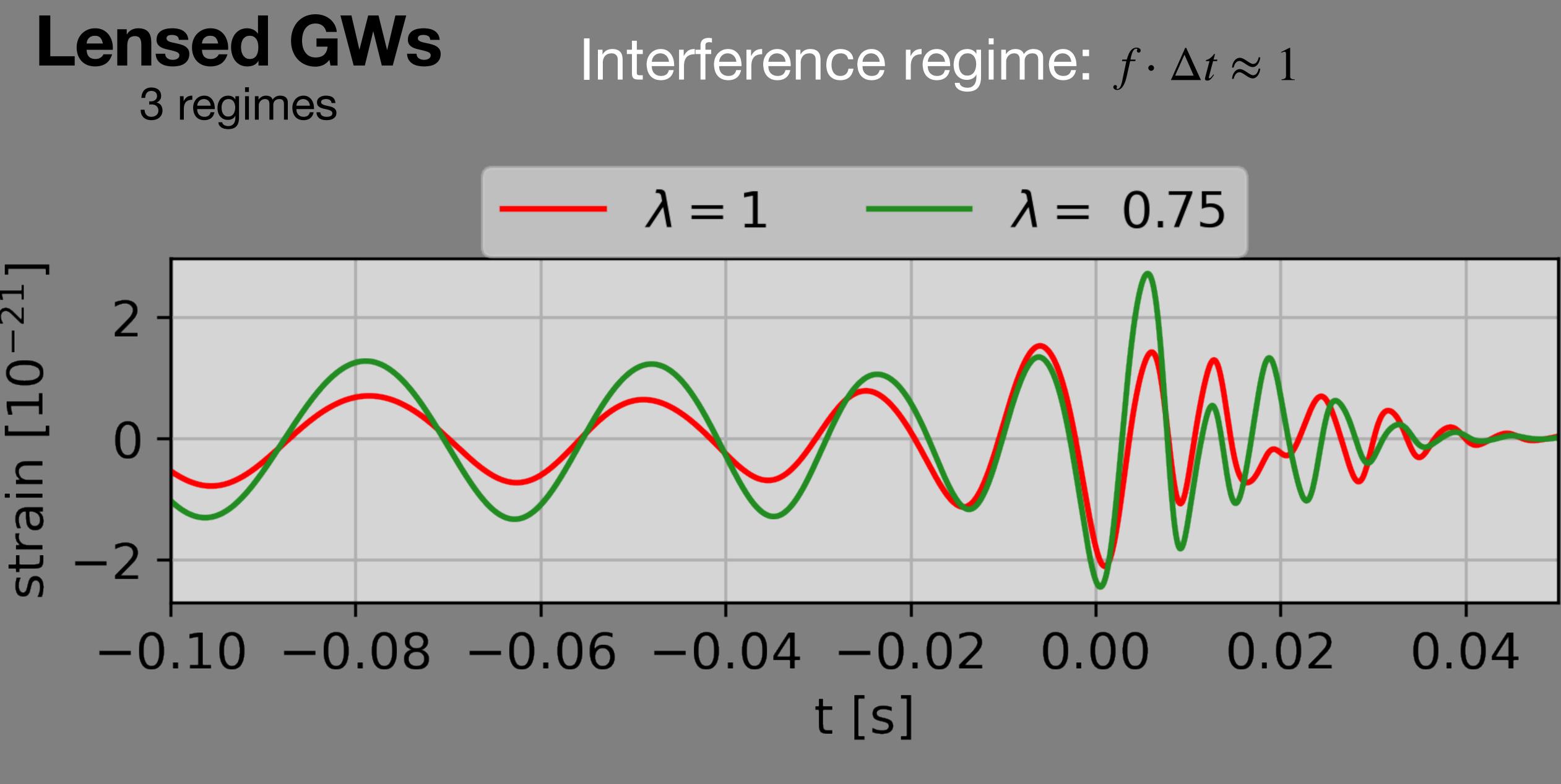
Lensed GWs Wave optics

q = 1

 10^{-16} characteristic strain [$\sqrt{4f^2}| ilde{h}(f)|^2$] 10^{-17} 10^{-18}

 10^{1}

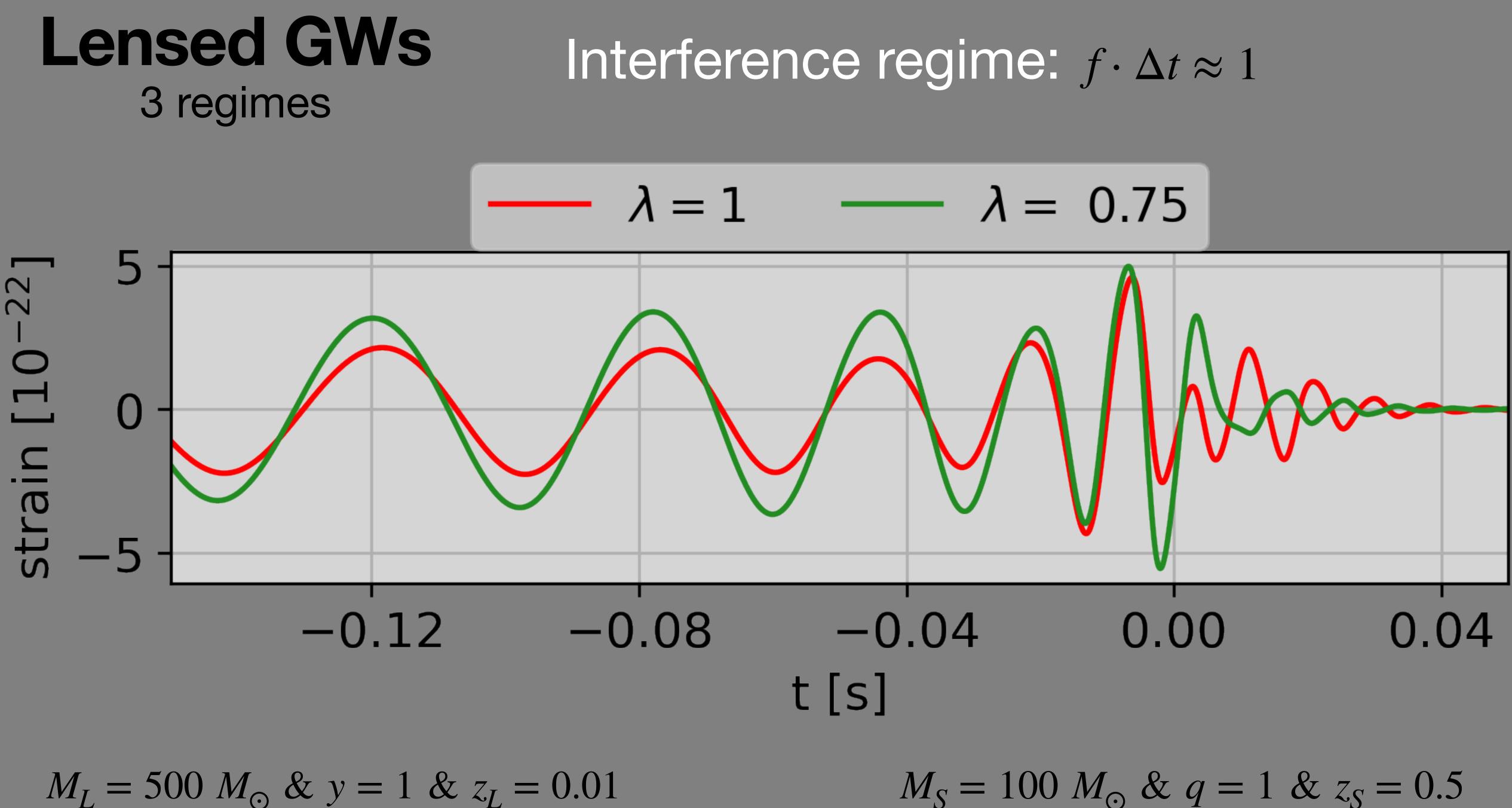




 $M_L = 500 \ M_{\odot} \& y = 1 \& z_L = 0.01$

 $M_S = 100 \ M_{\odot} \& q = 1 \& z_S = 0.1$



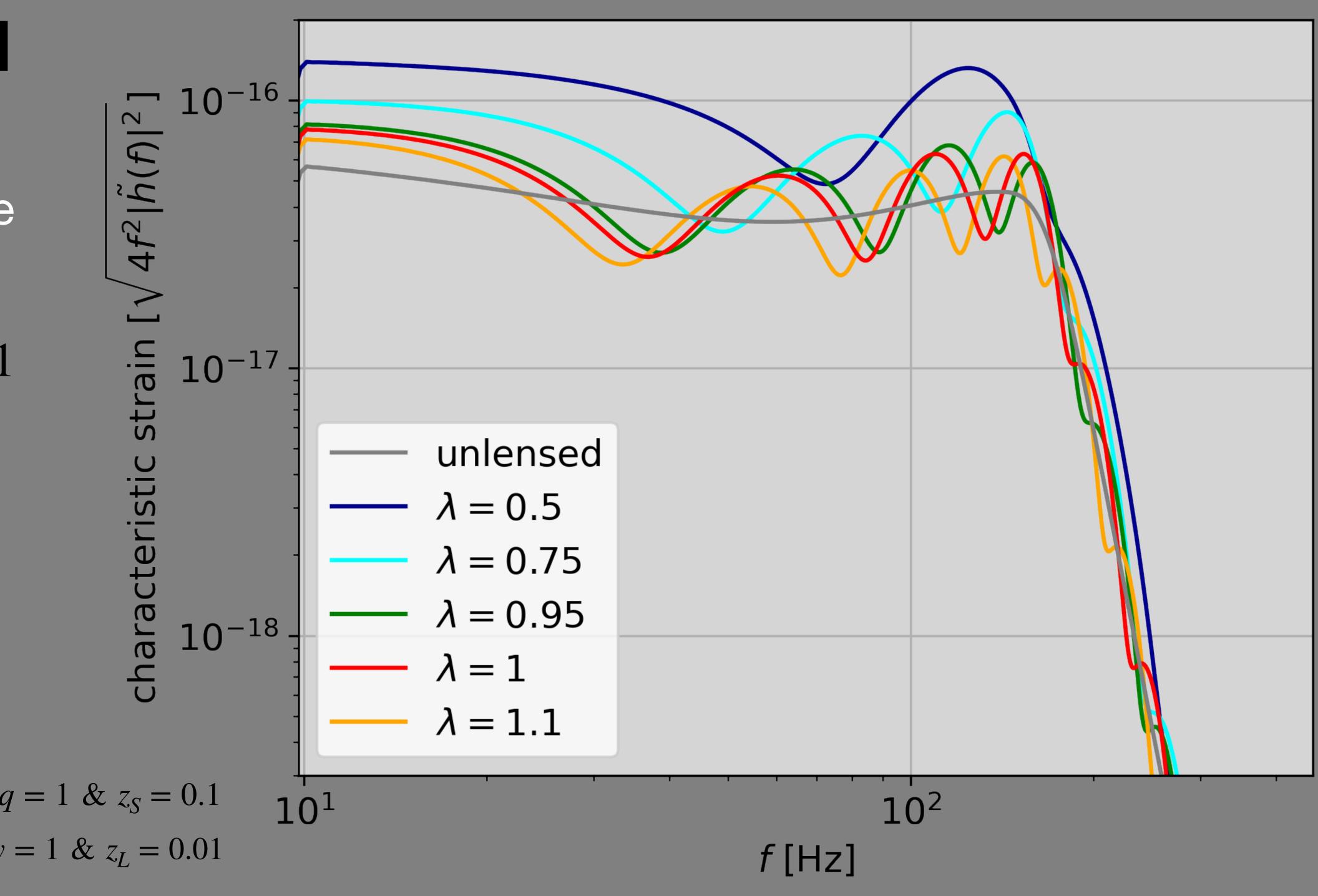


 $M_L = 500 \ M_{\odot} \& y = 1 \& z_L = 0.01$

Lensed GWs

Interference regime

 $^{\circ}f\cdot\Delta t\approx1$



 $M_S = 100 \ M_{\odot} \& q = 1 \& z_S = 0.1$ $M_L = 500 \ M_{\odot} \& y = 1 \& z_L = 0.01$

S/N - template matching The second second second in a static second second second and the second in a static second second second second Antonio and in the second second in and in the second second second second second second second second second s

Quantitative analysis

Signal-to-Noise ratio

 $\rho = \frac{\left(s \mid h_T\right)}{\sqrt{(h_T \mid h_T)}} \approx \frac{\left(h \mid h_T\right)}{\sqrt{(h_T \mid h_T)}}$

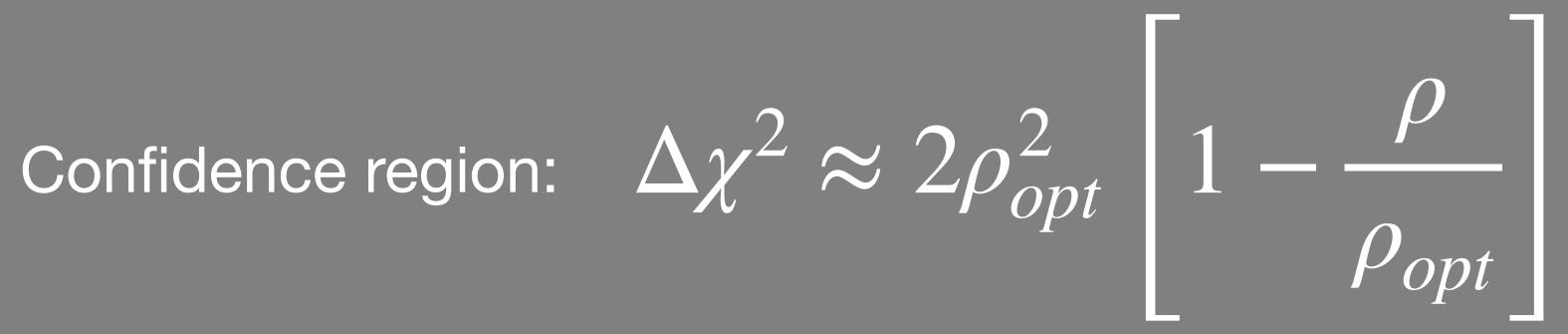
M. Maggiore, Gravitational Waves: Volume 1: Theory and Experiments, Gravitational Waves (OUP Oxford, 2008)

• s(t) = h(t) + n(t)

• Inner product:

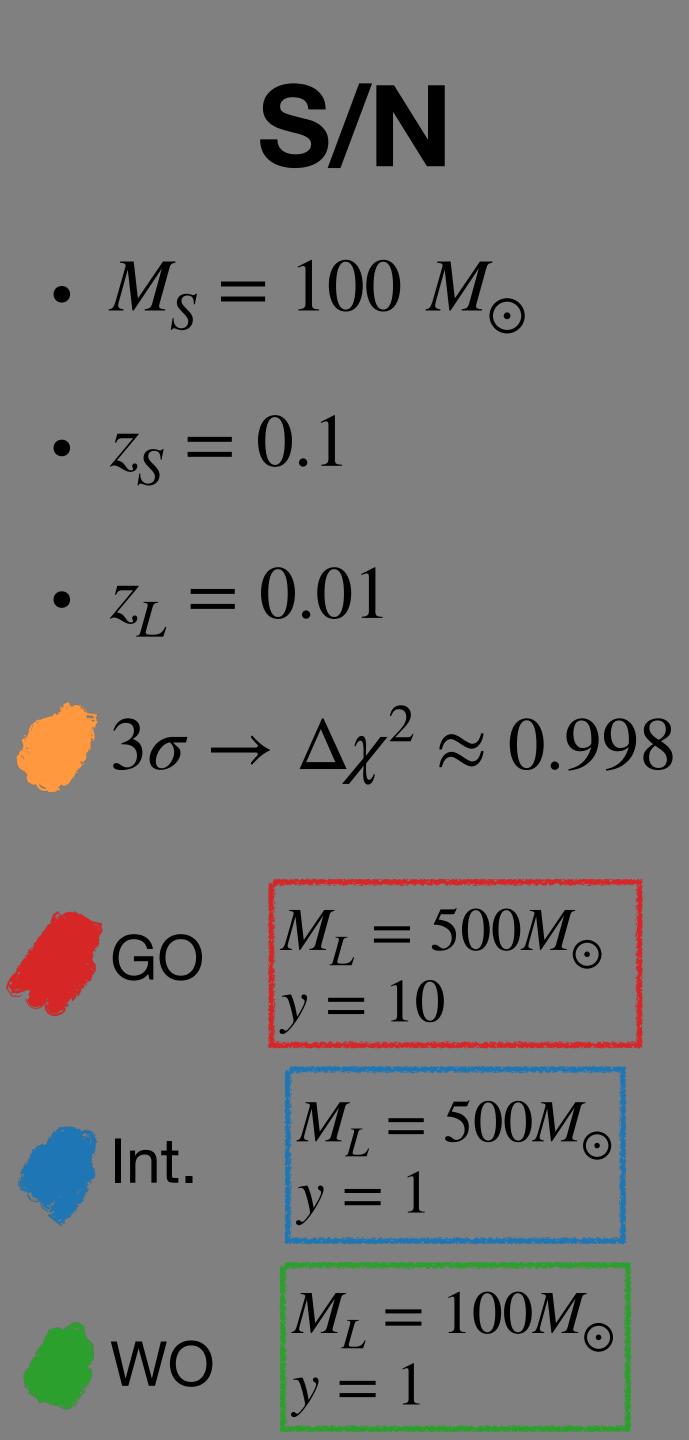
$$(a \mid b) = 4 \operatorname{Re} \left[\int_{0}^{\infty} \frac{\tilde{a}(f) \cdot \tilde{b}^{*}(f)}{S_{n}(f)} df \right]$$

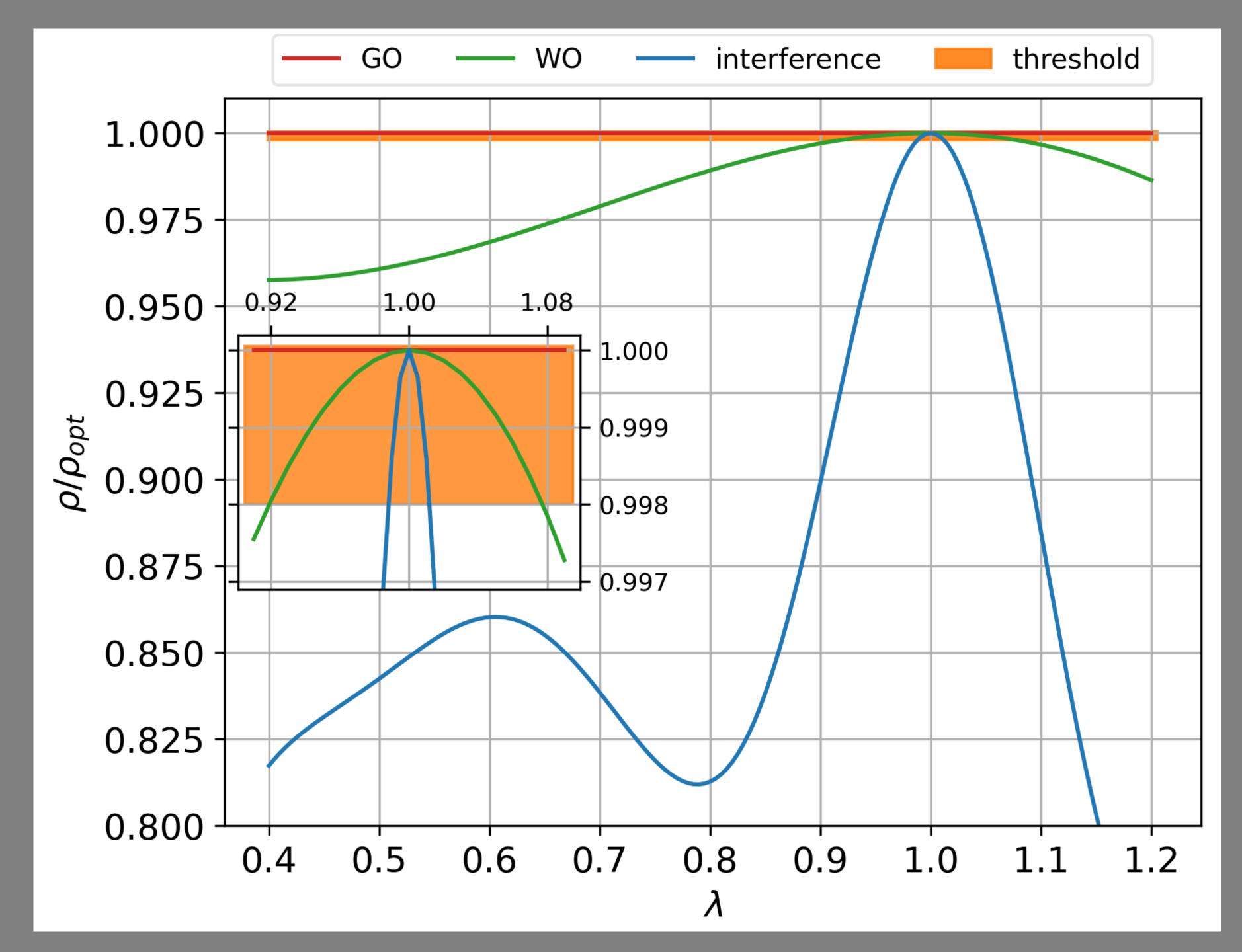
• $S_n(f)$ - (single-sided) power spectral density (O3 Ligo-L1)



 $3\sigma \rightarrow \Delta \chi^2 \approx 11.8$

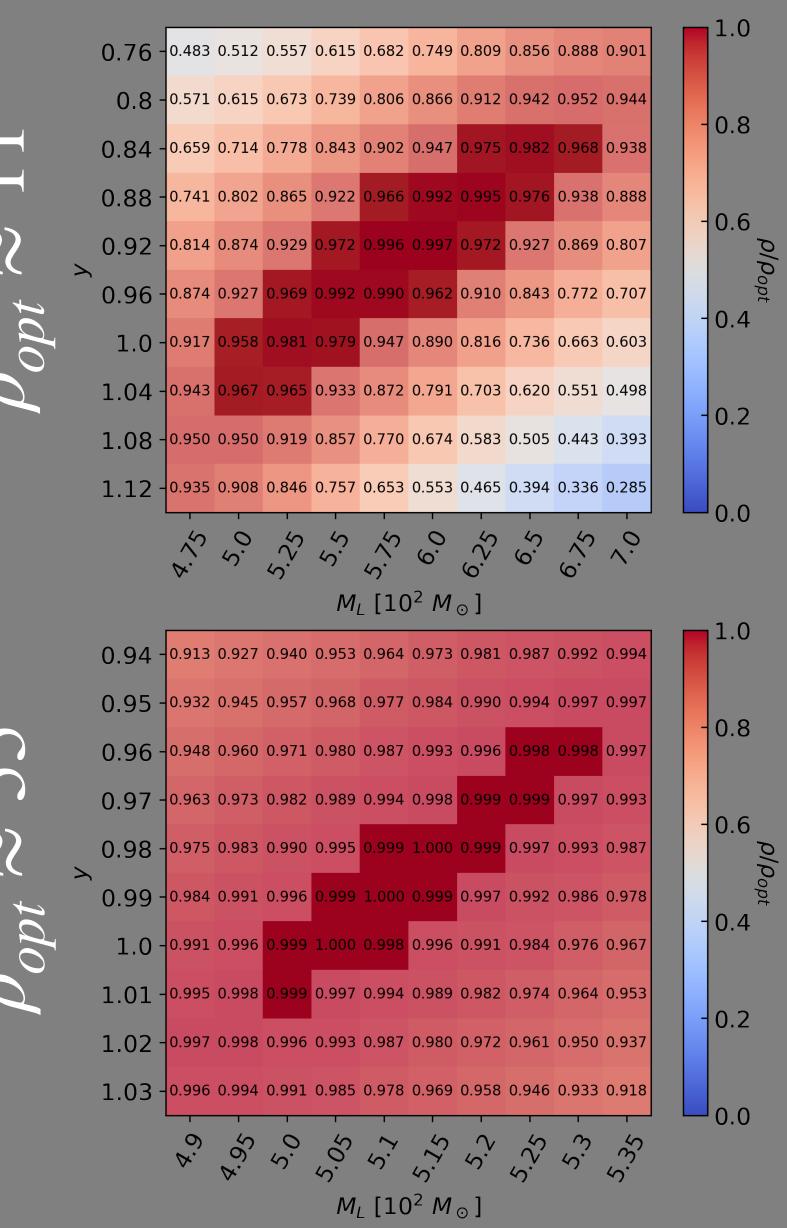






 $\lambda_{min} = 0.93$

S/



 $\lambda_{min} = 0.99$

0.85 - 0.54 0.54 0.57 0.63 0.71 0.80 0.88 0.93 0.95 0.94 0.9 - 0.59 0.61 0.67 0.74 0.83 0.91 0.96 0.98 0.96 0.91 **0.95** - 0.65 0.69 0.76 0.84 0.92 **0.98 1.00 0.97** 0.92 0.85 **1.0** - 0.70 0.76 0.84 0.92 **0.98 1.00 0.98** 0.92 0.83 0.76 **1.05** - 0.76 0.83 0.90 **0.97 1.00 0.98** 0.91 0.82 0.74 0.67 \rightarrow **1.1** - 0.81 0.88 0.95 0.99 0.98 0.91 0.82 0.72 0.64 0.59 **1.15** 0.86 0.93 0.97 0.98 0.92 0.82 0.71 0.61 0.55 0.50 **1.2** - 0.90 0.95 0.97 0.93 0.83 0.71 0.60 0.52 0.46 0.40 **1.25** - 0.93 0.96 0.94 0.85 0.73 0.60 0.50 0.43 0.36 0.28 1.3 - 0.94 0.95 0.88 0.76 0.62 0.50 0.42 0.34 0.25 0.15 $M_L [10^2 M_{\odot}]$

 $\lambda = 1$

1.0

0.8

-0.6

-0.4

0.2

0.0

1.0

0.8

0.6

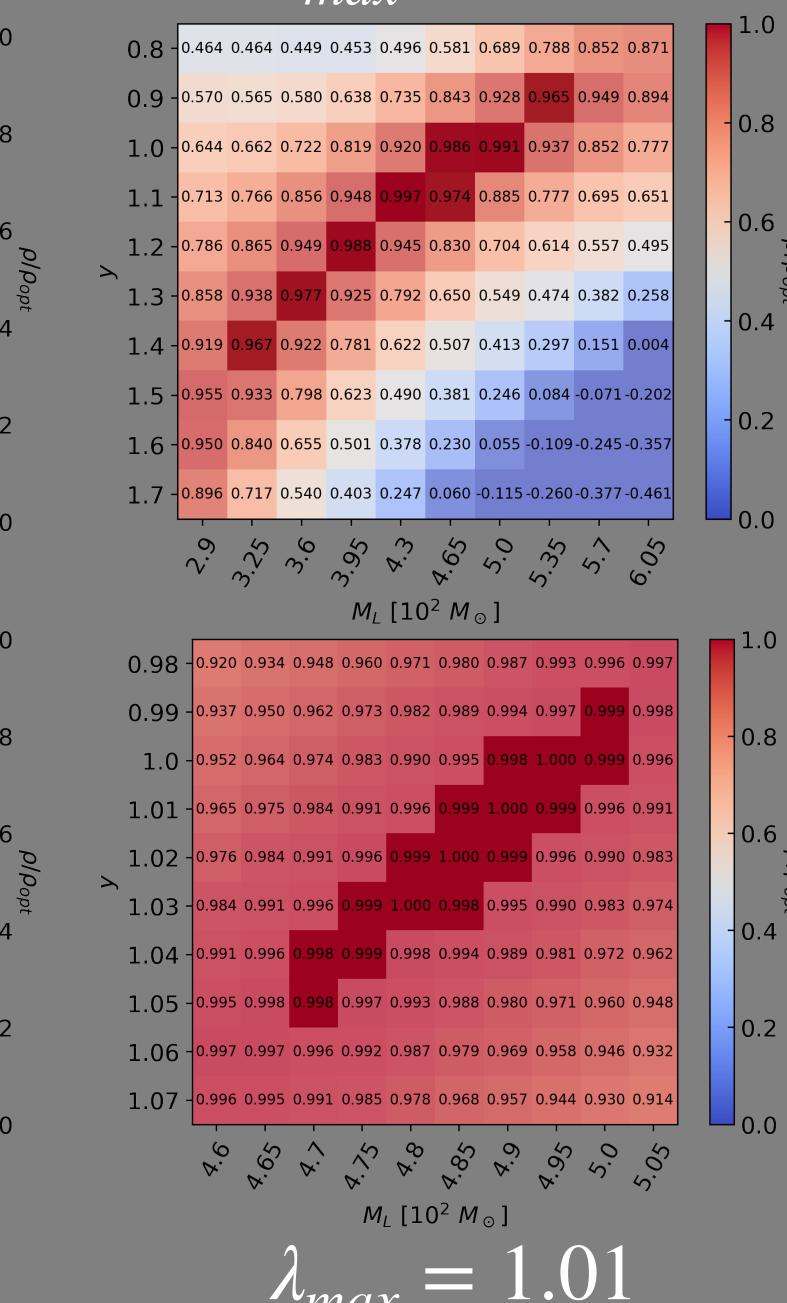
0.4

-0.2

0.0

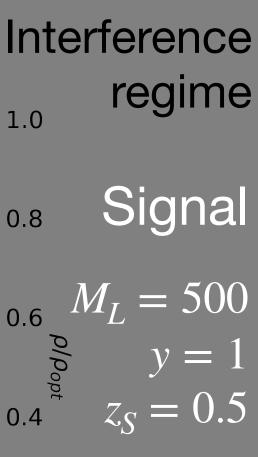
0.95 - 0.901 0.919 0.936 0.952 0.965 0.976 0.984 0.990 0.994 0.994 0.96 0.920 0.938 0.953 0.966 0.978 0.986 0.992 0.996 0.996 0.995 **0.97** - 0.938 0.954 0.967 0.978 0.987 0.994 0.997 **0.998** 0.996 0.992 **0.98** - 0.954 0.967 0.979 0.988 0.994 **0.998 0.999** 0.997 0.993 0.986 **0.99** - 0.967 0.979 0.988 0.995 **0.999 1.000 0.998** 0.994 0.987 0.978 **1.0** - 0.978 0.987 0.994 0.999 1.000 0.999 0.994 0.988 0.979 0.967 **1.01** - 0.986 0.994 0.998 1.000 0.999 0.995 0.988 0.979 0.967 0.954 **1.02** 0.992 0.997 **0.999 0.998** 0.995 0.988 0.979 0.968 0.954 0.939 **1.03** 0.996 0.998 0.998 0.994 0.988 0.979 0.968 0.954 0.939 0.922 **1.04** - 0.997 0.997 0.994 0.988 0.979 0.968 0.954 0.939 0.921 0.903 4. 2°

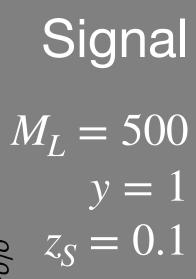
$t_{max} = 1.03$

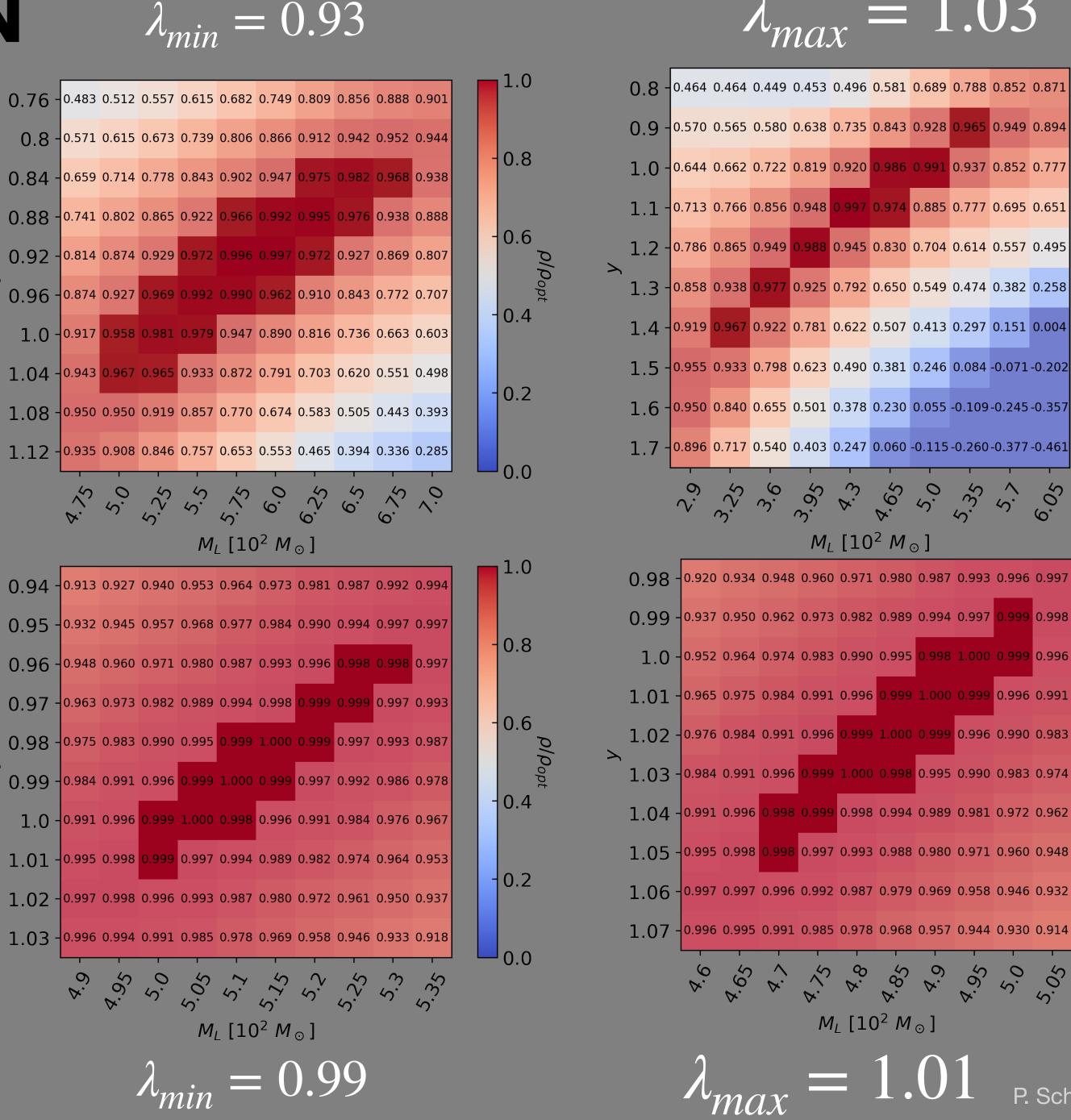


тах

5.00 5.2 5.2 5.2 5.2 5.2 5.2 5.2 5.2 5.0 $M_L [10^2 M_{\odot}]$







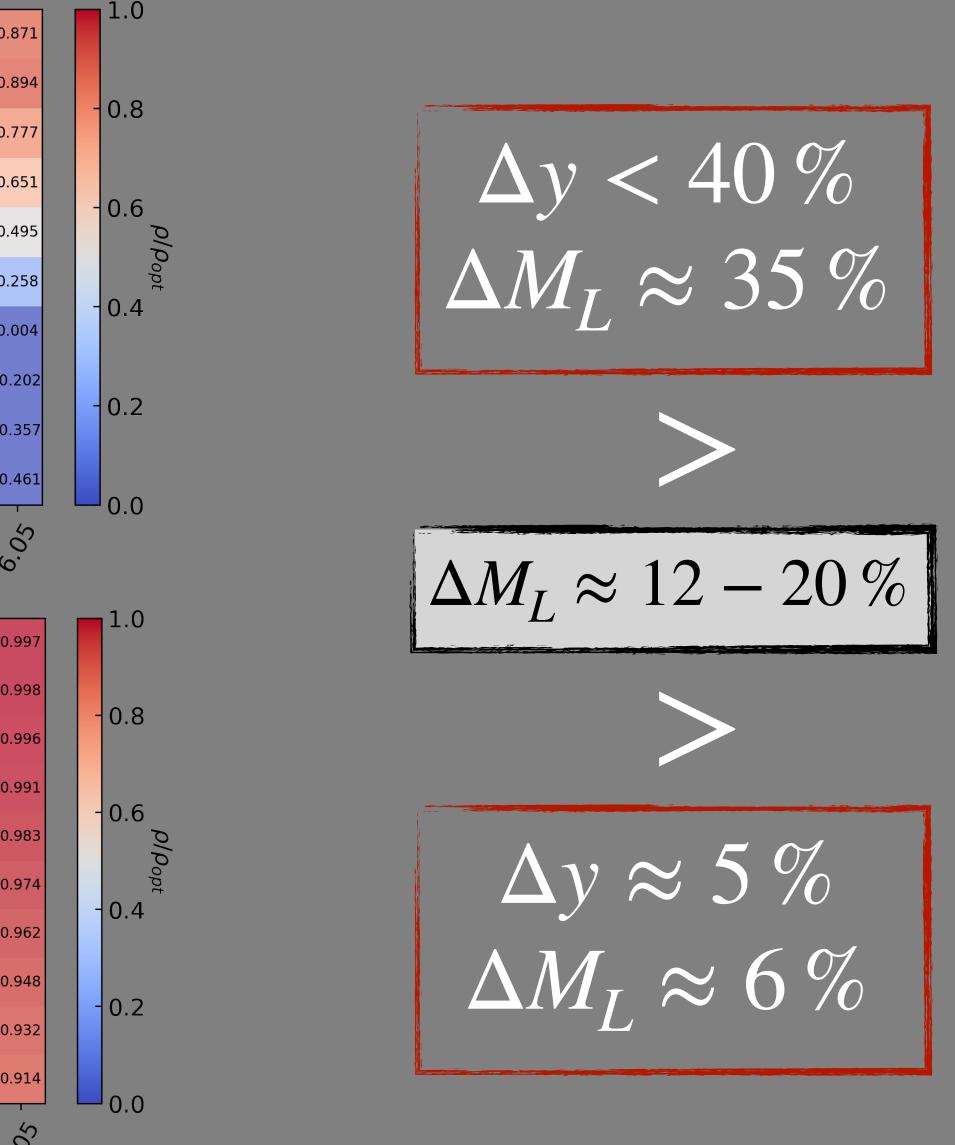
1.02 -

Q.9

S/N

opt

Interference regime



 $\Lambda_{max} = 1.03$

0.9 - 0.570 0.565 0.580 0.638 0.735 0.843 0.928 0.965 0.949 0.894 **1.0** - 0.644 0.662 0.722 0.819 0.920 0.986 0.991 0.937 0.852 0.777 **1.1** - 0.713 0.766 0.856 0.948 0.997 0.974 0.885 0.777 0.695 0.651 **1.2** - 0.786 0.865 0.949 0.988 0.945 0.830 0.704 0.614 0.557 0.495 **1.3** - 0.858 0.938 0.977 0.925 0.792 0.650 0.549 0.474 0.382 0.258 **1.4** - 0.919 0.967 0.922 0.781 0.622 0.507 0.413 0.297 0.151 0.004 **1.5** - 0.955 0.933 0.798 0.623 0.490 0.381 0.246 0.084 -0.071 -0.202 **1.6** - 0.950 0.840 0.655 0.501 0.378 0.230 0.055 -0.109-0.245-0.357 **1.7** - 0.896 0.717 0.540 0.403 0.247 0.060 -0.115 -0.260 -0.377 -0.461 $M_L [10^2 M_{\odot}]$ 0.98 -0.920 0.934 0.948 0.960 0.971 0.980 0.987 0.993 0.996 0.997 0.99 -0.937 0.950 0.962 0.973 0.982 0.989 0.994 0.997 0.999 0.998 **1.0** - 0.952 0.964 0.974 0.983 0.990 0.995 0.998 1.000 0.999 0.996 **1.01** - 0.965 0.975 0.984 0.991 0.996 **0.999 1.000 0.999** 0.996 0.991 **1.02** - 0.976 0.984 0.991 0.996 **0.999 1.000 0.999** 0.996 0.990 0.983 **1.03** - 0.984 0.991 0.996 **0.999 1.000 0.998** 0.995 0.990 0.983 0.974 **1.04** - 0.991 0.996 **0.998 0.999** 0.998 0.994 0.989 0.981 0.972 0.962 **1.05** - 0.995 0.998 **0.998** 0.997 0.993 0.988 0.980 0.971 0.960 0.948 1.06 - 0.997 0.997 0.996 0.992 0.987 0.979 0.969 0.958 0.946 0.932

 M_L [10² M_{\odot}]

P. Schneider and D. Sluse, Astron. Astrophys.559, A37(2013), arXiv:1306.0901 [astro-ph.CO]





Conclusions



- 1. We analysed how MSD behave in GW lensing 2. In the GO regime it can not be broken
- 3. In WO can be broken in some cases
- 4. In interference regime is broken

5. How well it is broken depends on the strength of the signal and sensitivity of detectors. Nowadays we might have up to $\Delta y \approx 5\,\%\,$ and $\Delta M \approx 6\,\%\,$





THE END

