



Breaking the Mass Sheet Degeneracy with Gravitational Waves Interference in Lensing events

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Mass Sheet Degeneracy

Mass Sheet Degeneracy

E. E. Falco, M. V. Gorenstein, and I. I. Shapiro, ApJ 289, L1 (1985)

- Scalings of lens mass:

- $\kappa \rightarrow \kappa_\lambda = \lambda\kappa + (1 - \lambda)$

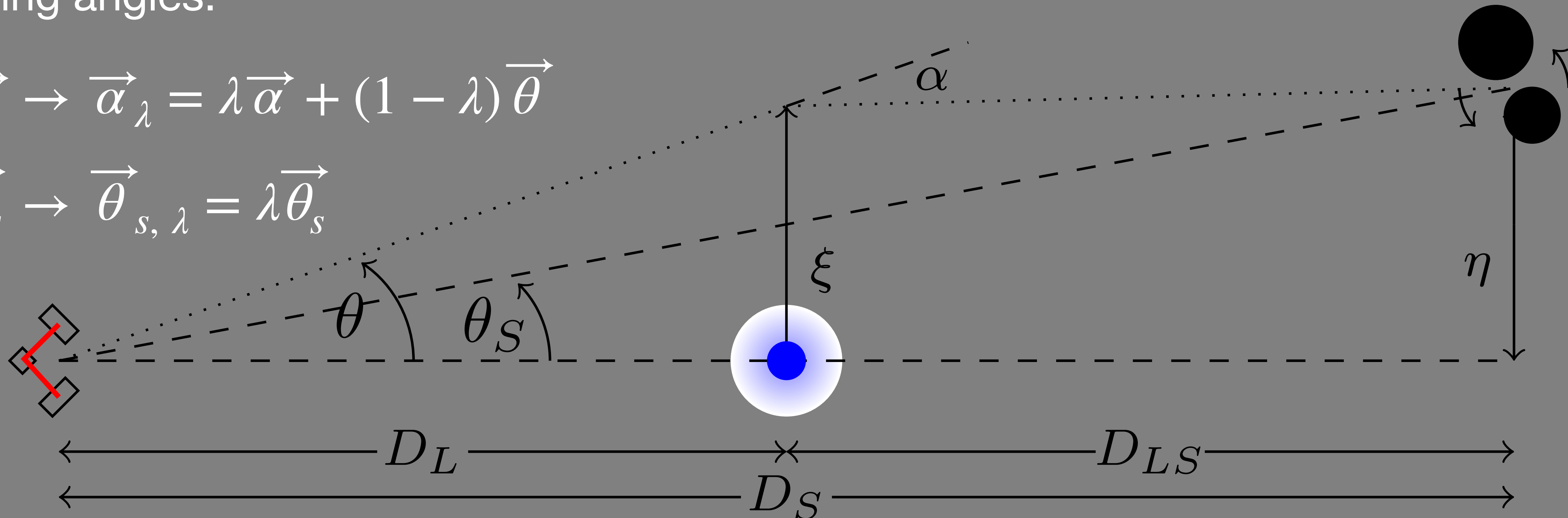
$$\kappa = \Sigma / \Sigma_{cr}$$

Σ - surface mass density

- Scaling angles:

- $\vec{\alpha} \rightarrow \vec{\alpha}_\lambda = \lambda\vec{\alpha} + (1 - \lambda)\vec{\theta}$

- $\vec{\theta}_s \rightarrow \vec{\theta}_{s,\lambda} = \lambda\vec{\theta}_s$



MSD

Why a problem?

- Observables are preserved!
- Problems: e.g. biased estimations of mass lens
- Biased estimation of cosmological parameter, e.g. H_0

Can we solve it?

- EM geometrical optics regime: multiple images; independent mass estimation of the lens (e.g. dynamics)
- EM wave optics regime: multiple lenses
- **In GW lensing: 1 image and 1 lens can break MSD!**

Gravitational Waves Lensing

Gravitational Lensing of Grav. Waves

NB: spherical symmetry!

- $\tilde{h}(f) \cdot F(f, \theta_s) = \tilde{h}_L(f)$

- $F(w, y) = -i w e^{i w y^2 / 2} \int_0^\infty dx x J_0(w x y) \exp \left\{ i w \left[\frac{1}{2} x^2 - \Psi(x) \right] \right\} \rightarrow F_\lambda$

- Where:

- $w = \frac{1 + z_L}{c} \frac{D_S D_L \theta_E^2}{D_{LS}} 2\pi f$

- $x = |\vec{x}| = |\vec{\theta} / \vec{\theta}_E|$

- $y = |\vec{y}| = |\vec{\theta}_s / \vec{\theta}_E| \rightarrow y_\lambda$

- J_0 - Bessel function of 0-th order

- Ψ - dimensionless effective lensing potential

Ψ_λ

T. T. Nakamura and S. Deguchi, Progress of Theoretical Physics Supplement 133, 137 (1999).

Lensed waveforms under mass-sheet transformation

Qualitative analysis

Lensed GWs

3 regimes

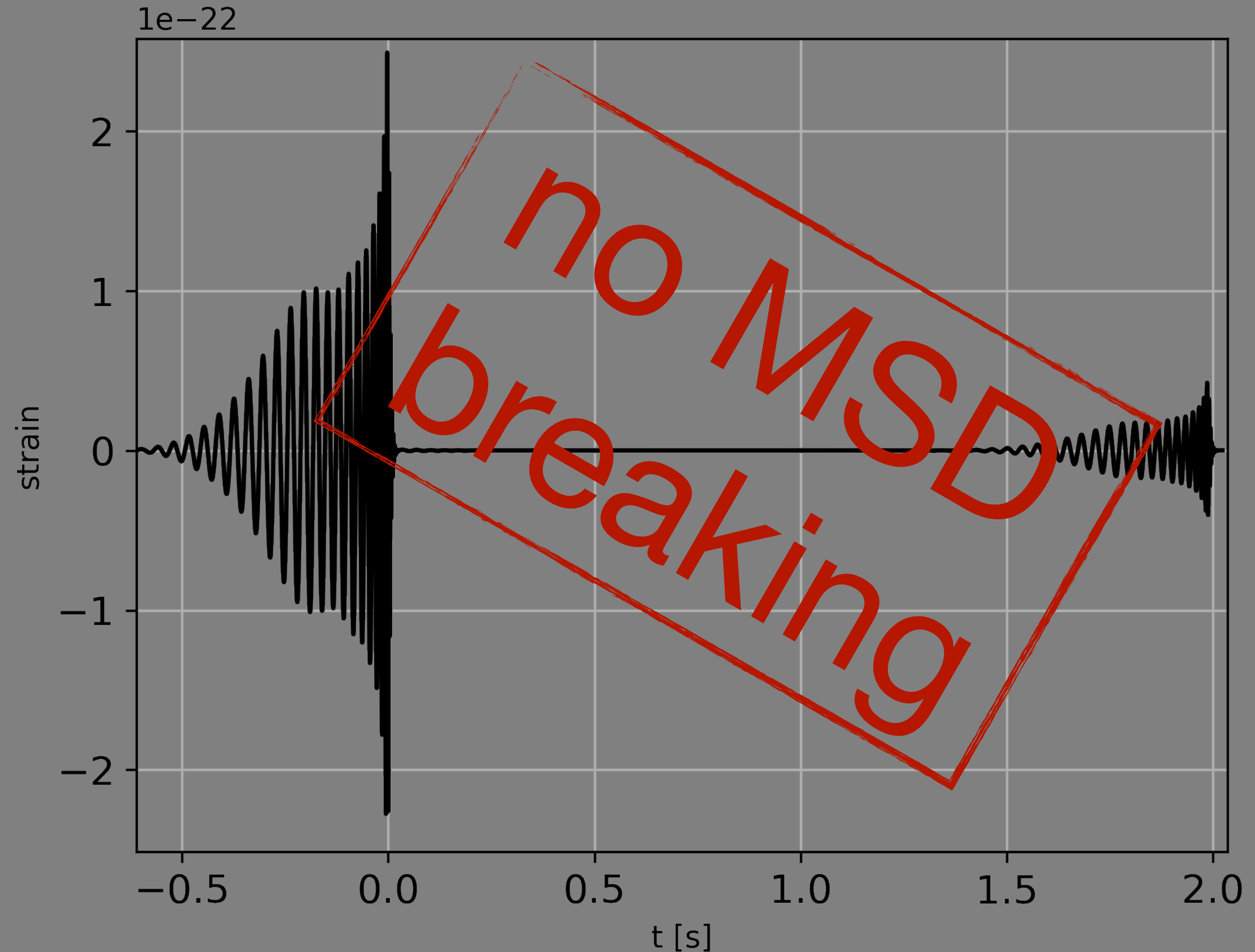
- Geometrical Optics

- $f \cdot \Delta t \gg 1$

- $M_L > 10^5 [(1 + z_L)f]^{-1}$

$$M_S = 60 M_\odot \quad - \quad z_S = 0.5$$

$$M_L = 10^4 M_\odot \quad - \quad z_L = 0.1 \quad - \quad y = 5$$



Lensed GWs

3 regimes

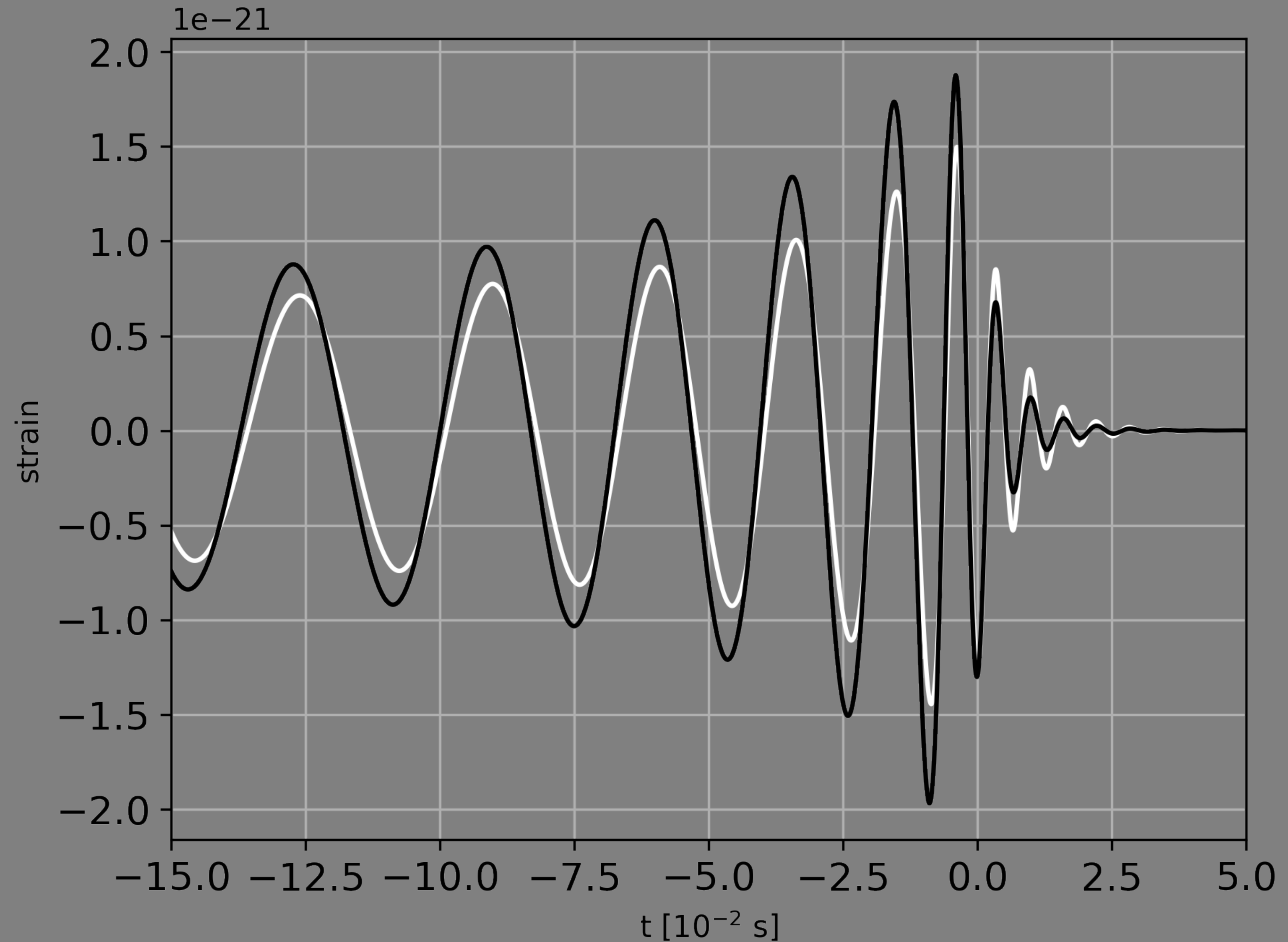
- Wave Optics

- $f \cdot \Delta t \lesssim 1$

- $M_L \leq 10^5 [(1 + z_L)f]^{-1}$

$$M_S = 100 M_\odot \quad - \quad z_S = 0.1$$

$$M_L = 100 M_\odot \quad - \quad z_L = 0.01$$



Unlensed



Lensed

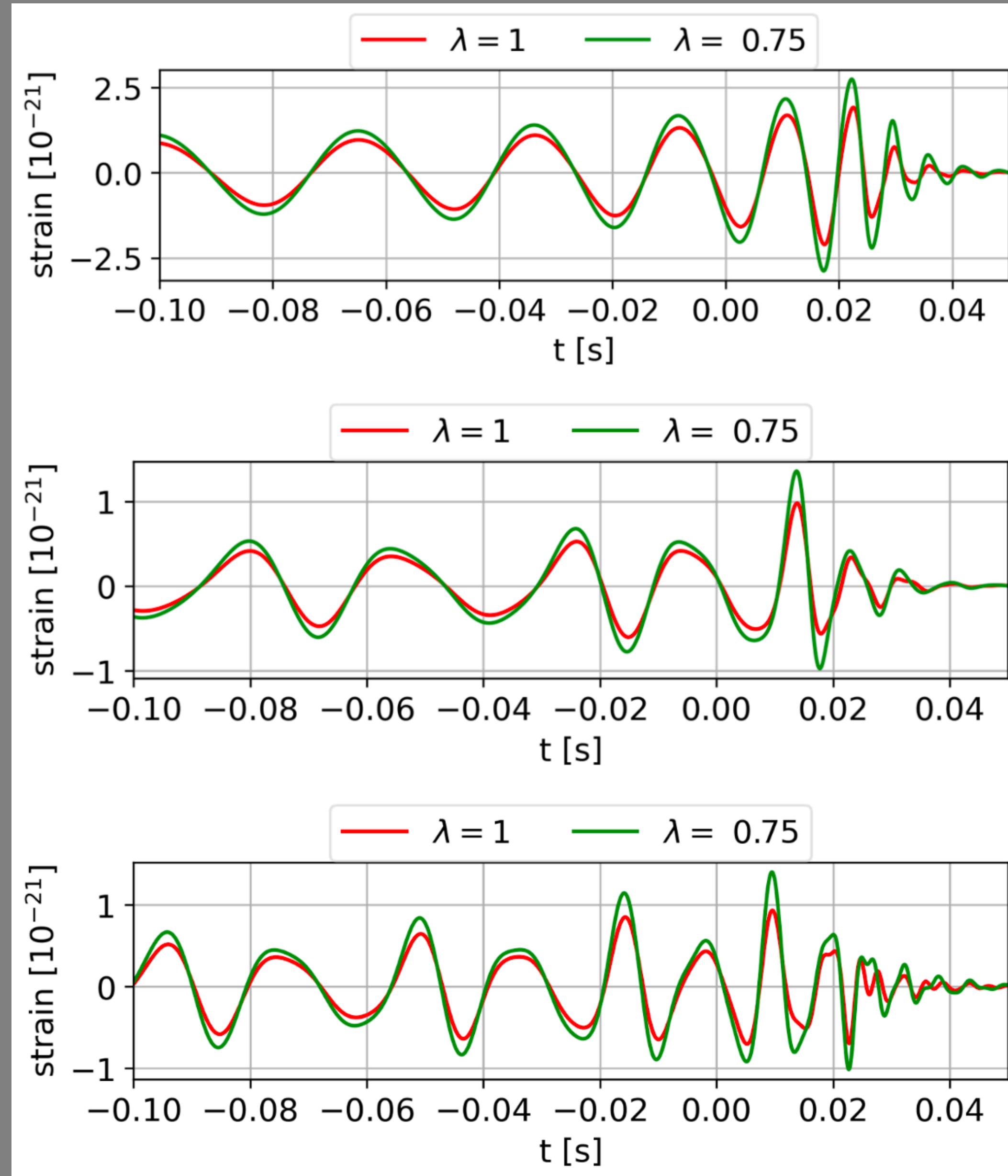
Lensed GWs

Wave optics

$$q = \frac{m_2}{m_1} = 1$$

$$q = 0.1$$

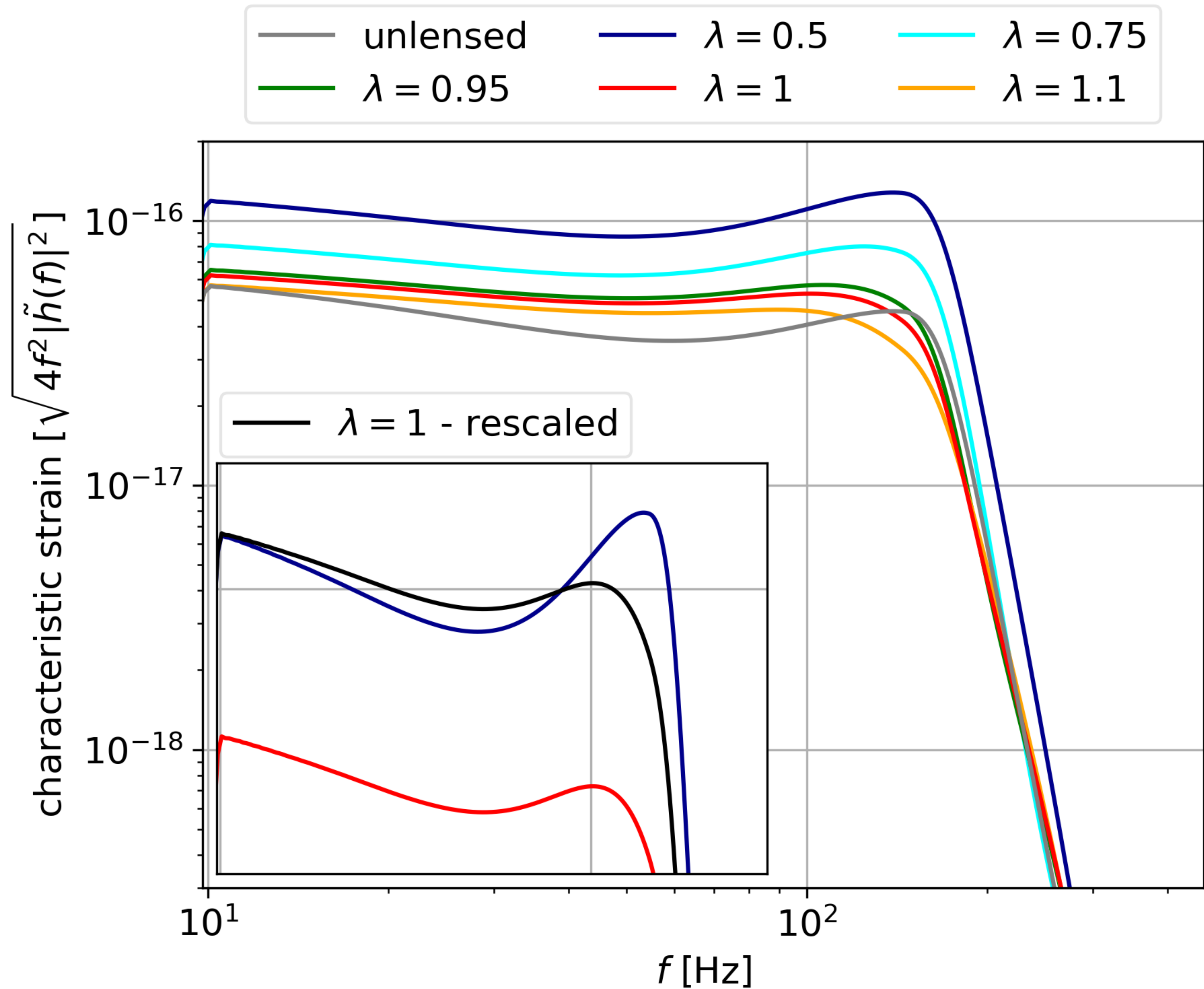
$$q = 0.1 \text{ \& } s_{1,2;z} = \{0.7, 0.2\}$$



Lensed GWs

Wave optics

$$q = 1$$

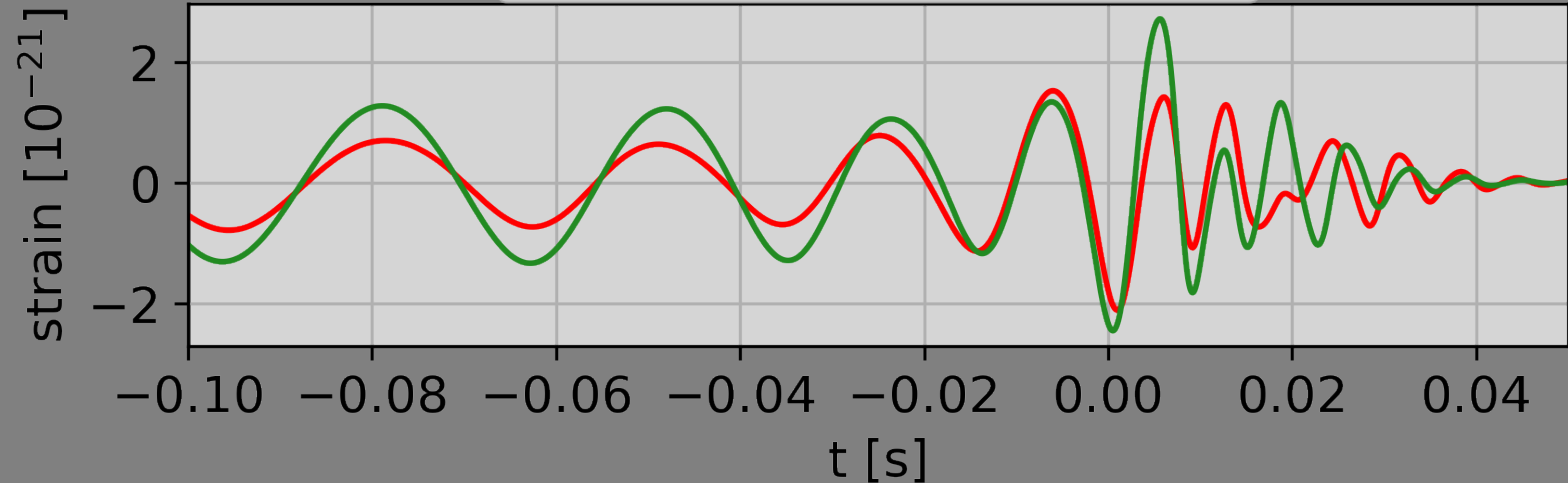


Lensed GWs

3 regimes

Interference regime: $f \cdot \Delta t \approx 1$

— $\lambda = 1$ — $\lambda = 0.75$



$$M_L = 500 M_\odot \ \& \ y = 1 \ \& \ z_L = 0.01$$

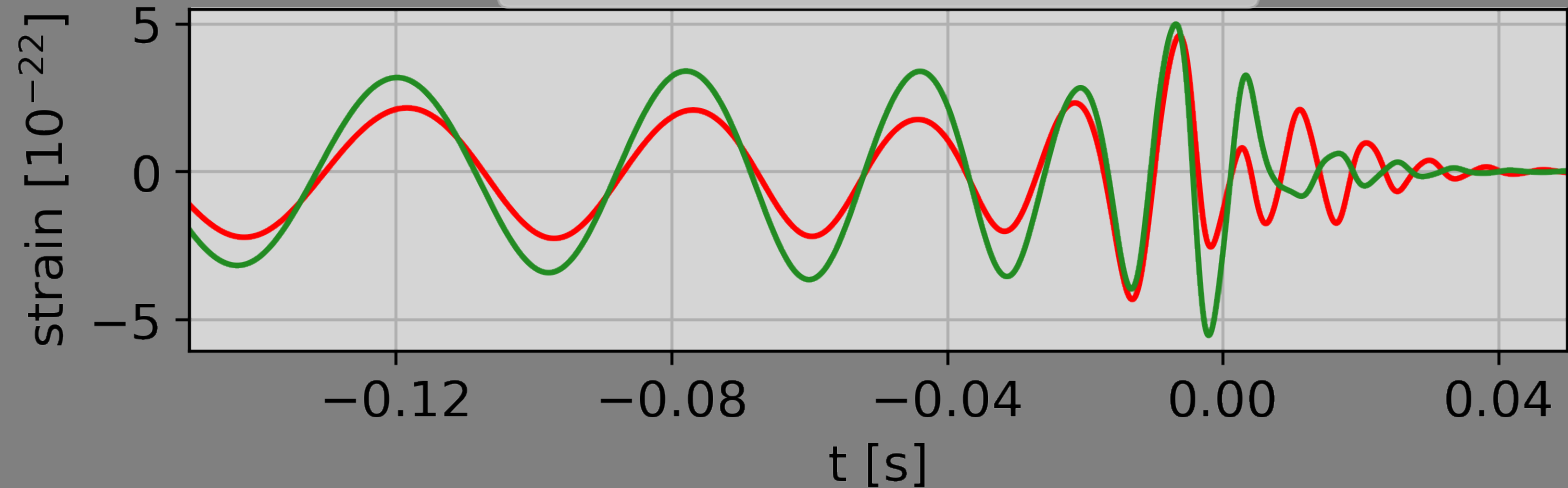
$$M_S = 100 M_\odot \ \& \ q = 1 \ \& \ z_S = 0.1$$

Lensed GWs

3 regimes

Interference regime: $f \cdot \Delta t \approx 1$

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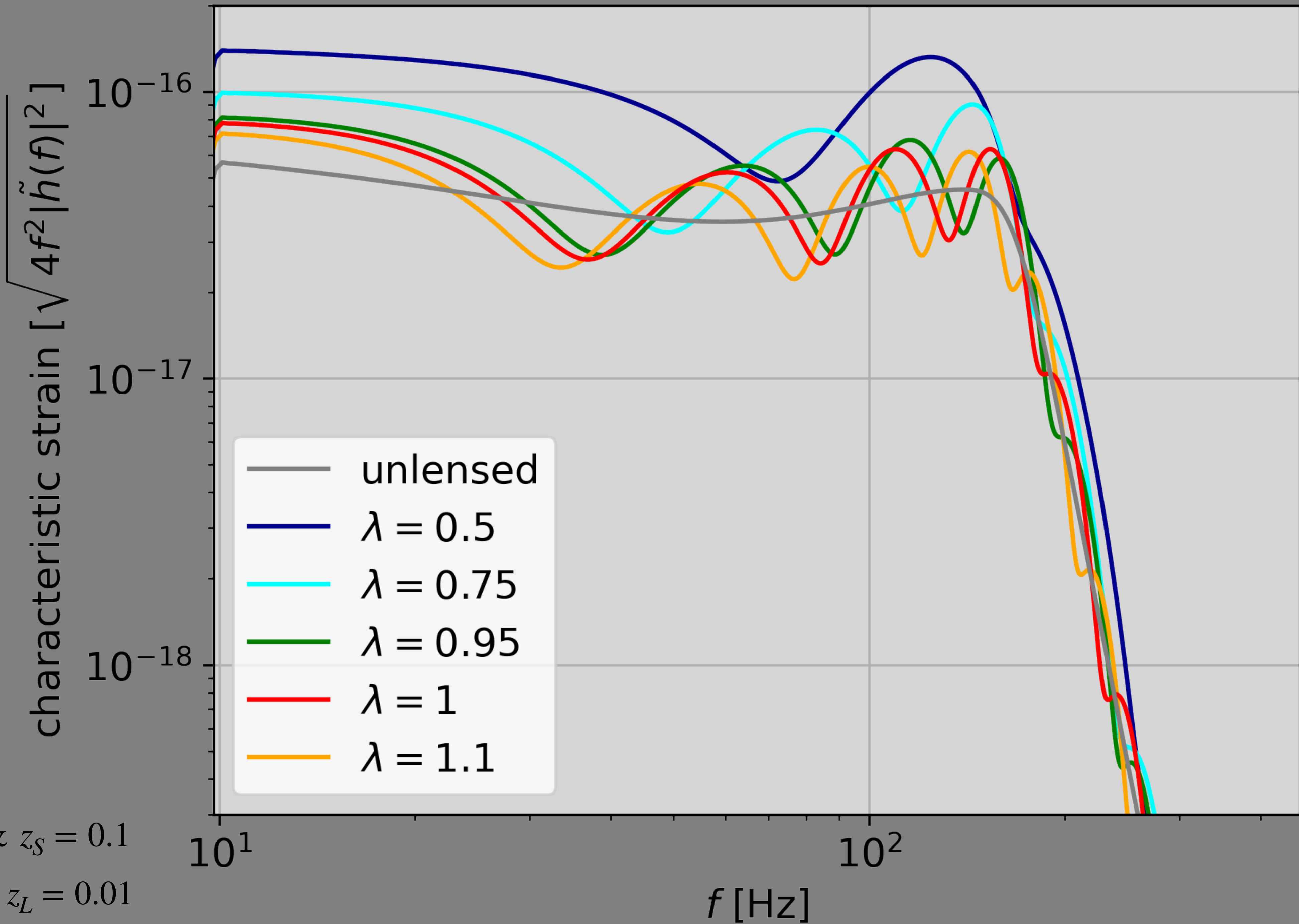
$$M_S = 100 M_\odot \ \& \ q = 1 \ \& \ z_S = 0.5$$

Lensed GWs

Interference
regime

$$\circ f \cdot \Delta t \approx 1$$

$M_S = 100 M_\odot$ & $q = 1$ & $z_S = 0.1$
 $M_L = 500 M_\odot$ & $y = 1$ & $z_L = 0.01$



S/N - template matching

Quantitative analysis

Signal-to-Noise ratio

$$\rho = \frac{(s | h_T)}{\sqrt{(h_T | h_T)}} \approx \frac{(h | h_T)}{\sqrt{(h_T | h_T)}}$$

- $s(t) = h(t) + n(t)$

- Inner product:

$$(a | b) = 4 \operatorname{Re} \left[\int_0^\infty \frac{\tilde{a}(f) \cdot \tilde{b}^*(f)}{S_n(f)} df \right]$$

- $S_n(f)$ - (single-sided) power spectral density (L1-O3-LIGO)

Confidence region: $\Delta\chi^2 \approx 2\rho_{opt}^2 \left[1 - \frac{\rho}{\rho_{opt}} \right]$ $3\sigma \rightarrow \Delta\chi^2 \approx 11.8$

S/N

- $M_S = 100 M_\odot$

- $z_S = 0.1$

- $z_L = 0.01$

 $3\sigma \rightarrow \Delta\chi^2 \approx 0.998$

 GO

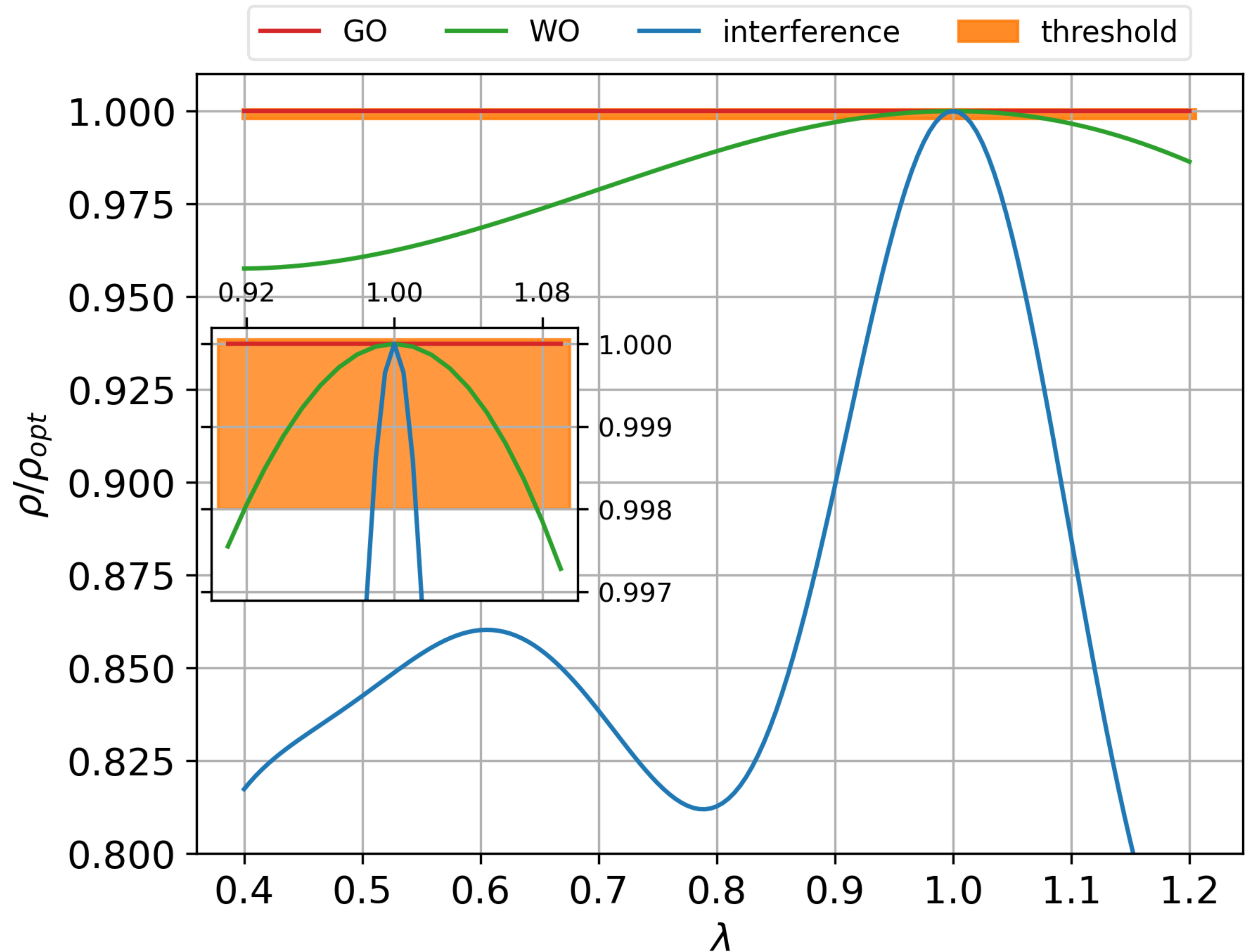
$M_L = 500M_\odot$
 $y = 10$

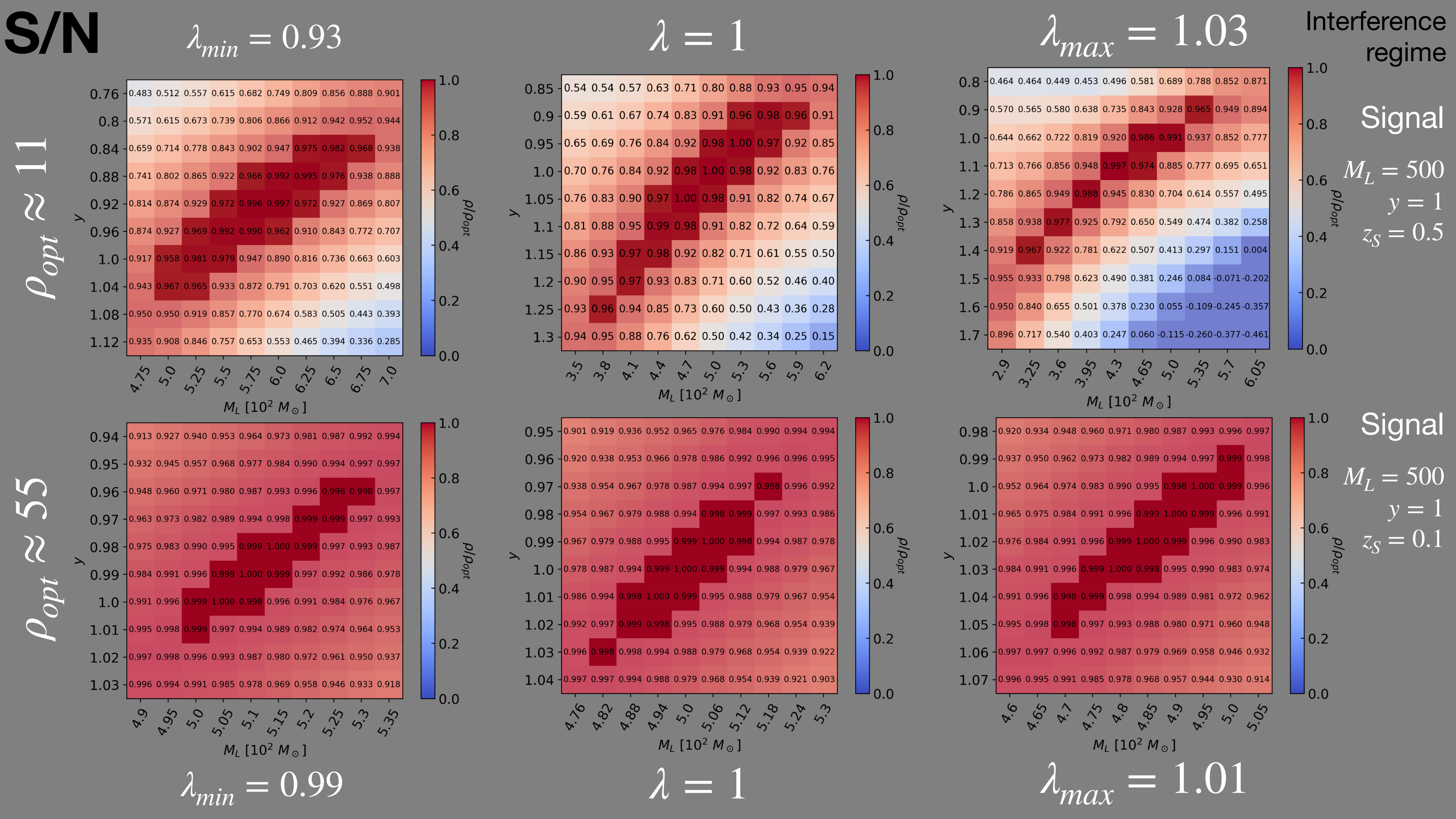
 Int.

$M_L = 500M_\odot$
 $y = 1$

 WO

$M_L = 100M_\odot$
 $y = 1$

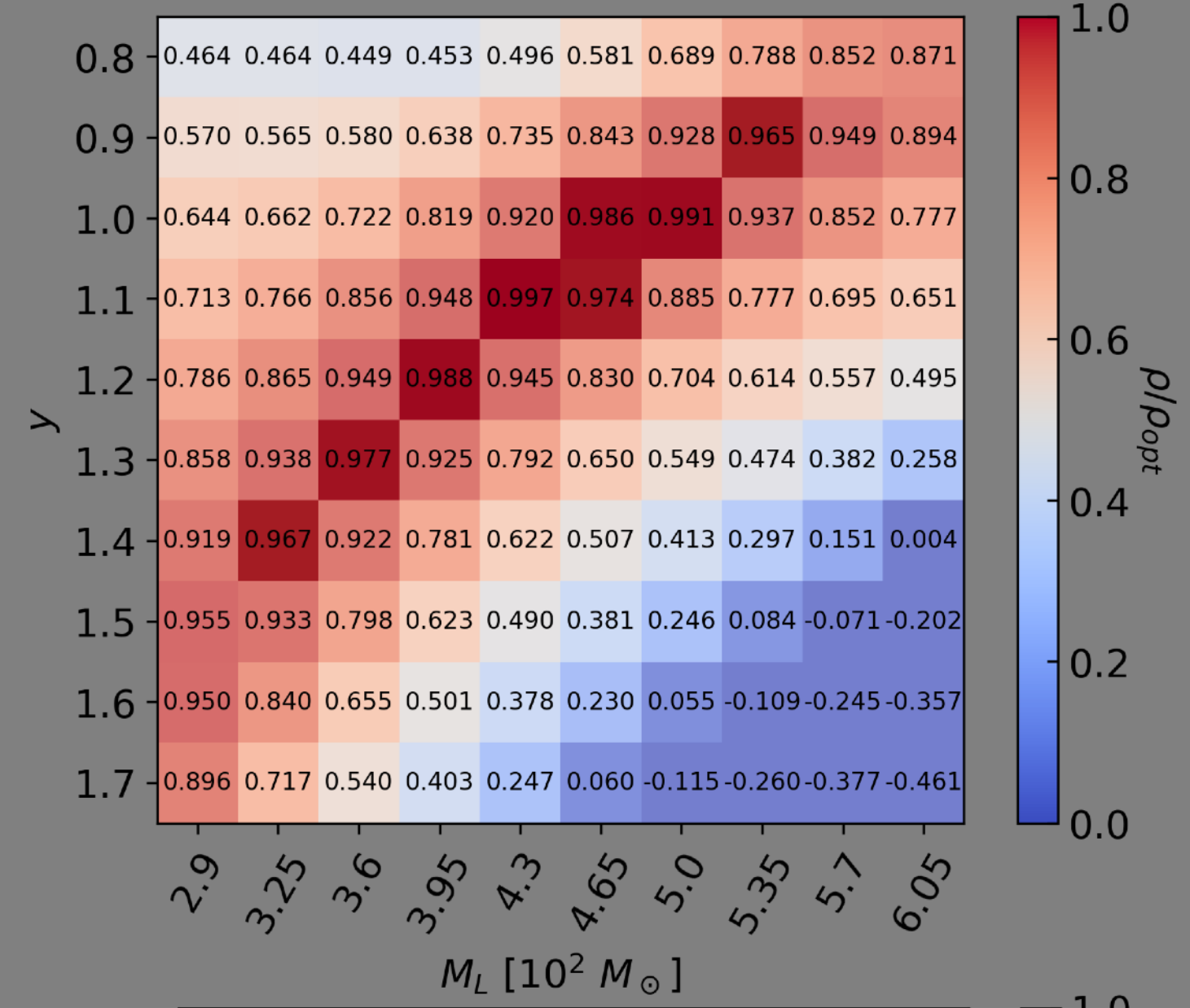
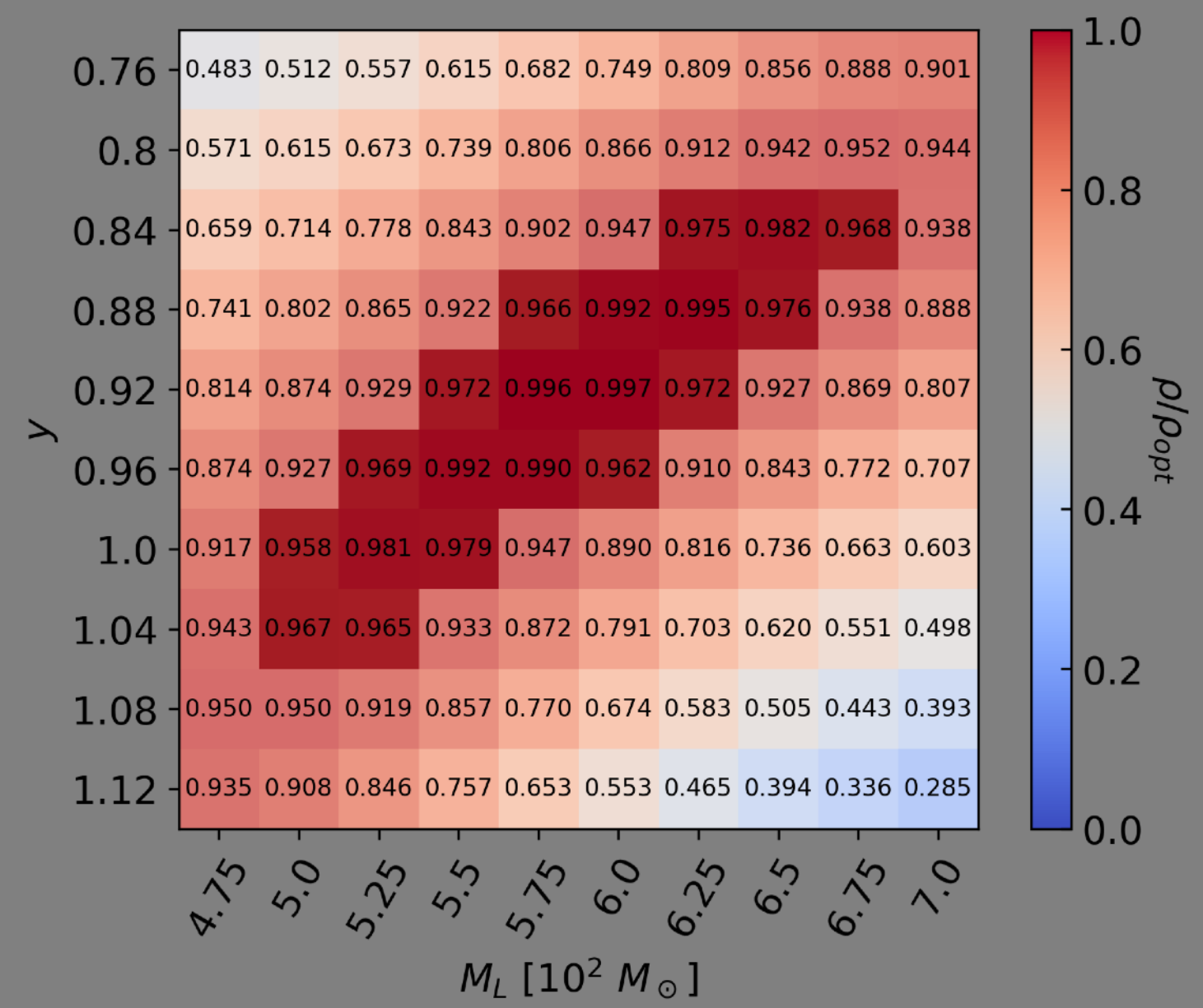




S/N

 $\lambda_{min} = 0.93$ $\lambda_{max} = 1.03$

Interference regime

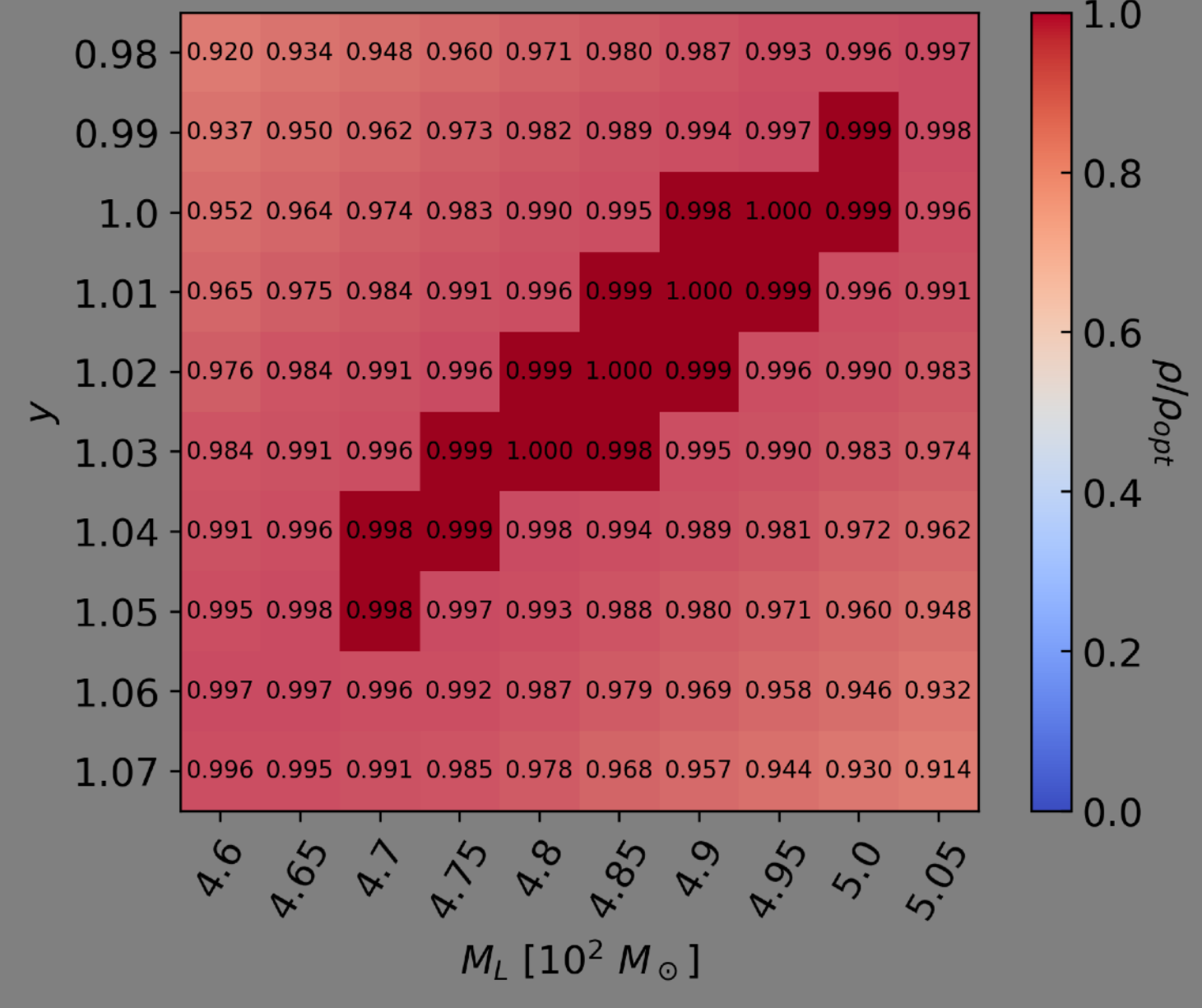
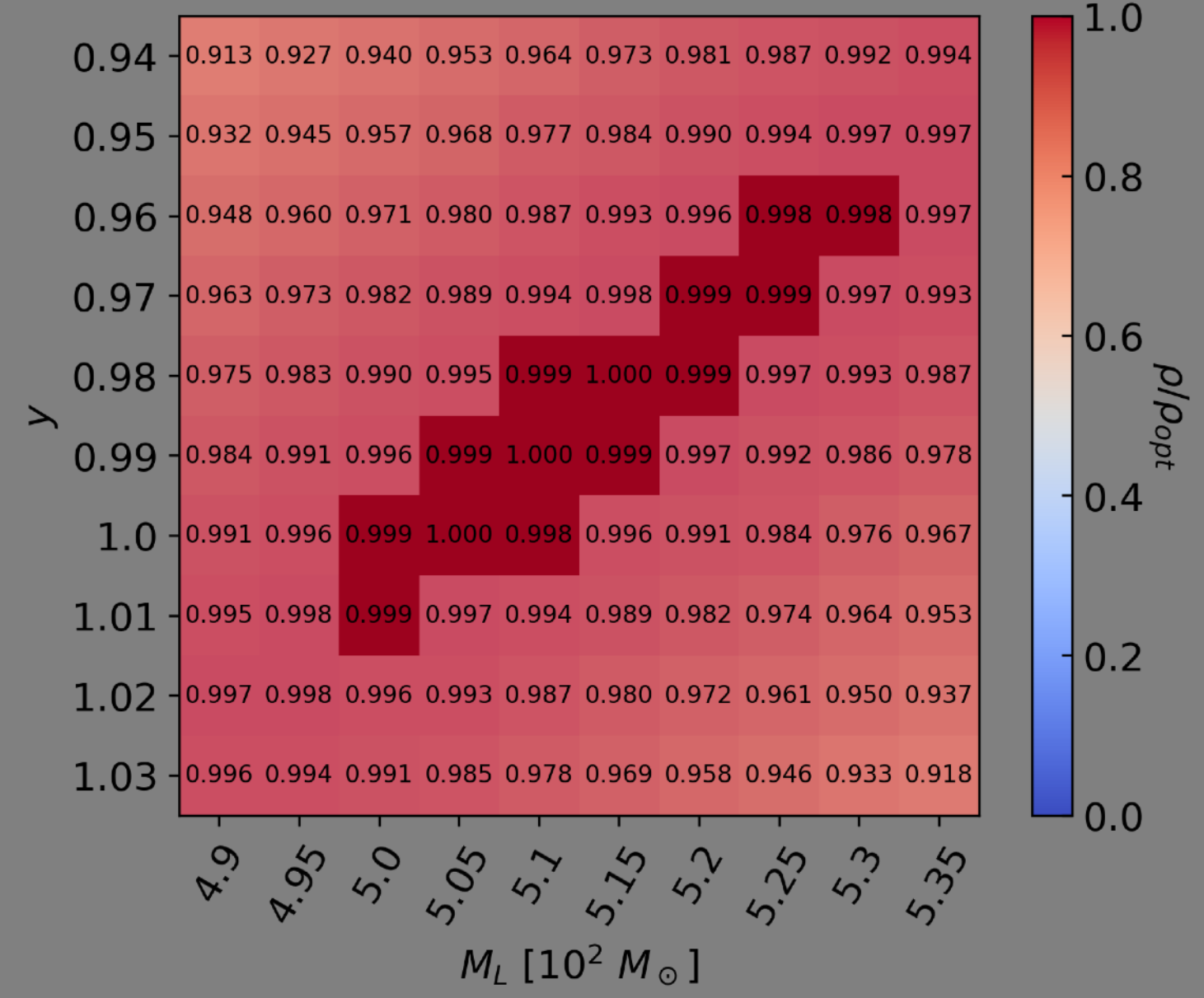
 $\rho_{opt} \approx 11$ 

$\Delta y < 40\%$
 $\Delta M_L \approx 35\%$

>

$\Delta M_L \approx 12 - 20\%$

>

 $\rho_{opt} \approx 55$ 

$\Delta y \approx 5\%$
 $\Delta M_L \approx 6\%$

 $\lambda_{min} = 0.99$ $\lambda_{max} = 1.01$

Conclusions

Conclusions

1. We analysed how MSD behave in GW lensing
2. In the GO regime it can not be broken
3. In WO can be broken in some cases
4. In interference regime is broken
5. How well it is broken depends on the strength of the signal and sensitivity of detectors. Nowadays we might have up to $\Delta y \approx 5\%$ and $\Delta M \approx 6\%$

More info