Wave Optics in Gravitational Wave Lensing

UBC - Gravity seminar

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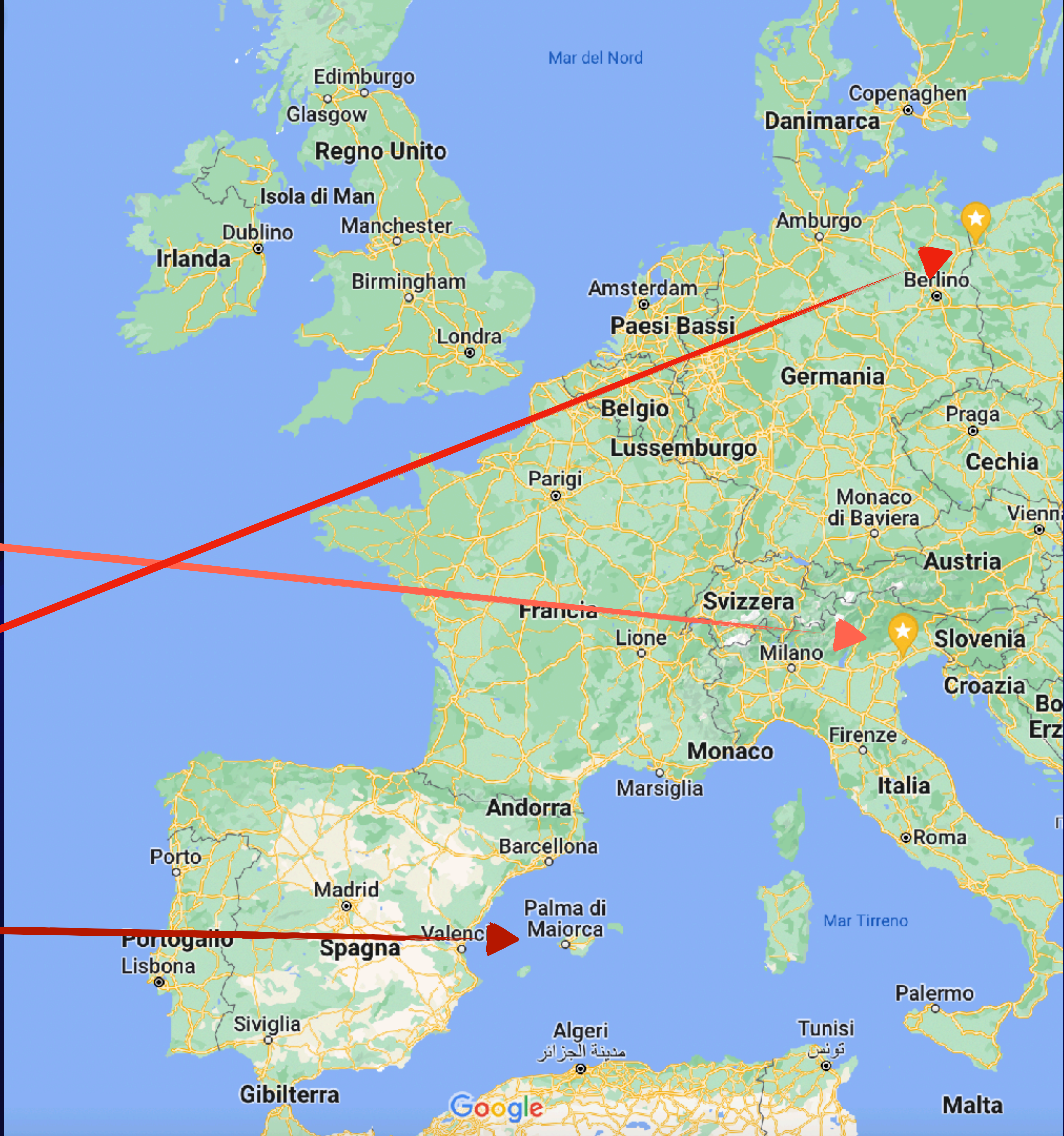
07.09.2023

For the curious

- Born in Treviso and graduated in Padova
- PhD in Szczecin, Poland
- PostDoc in Palma, Mallorca



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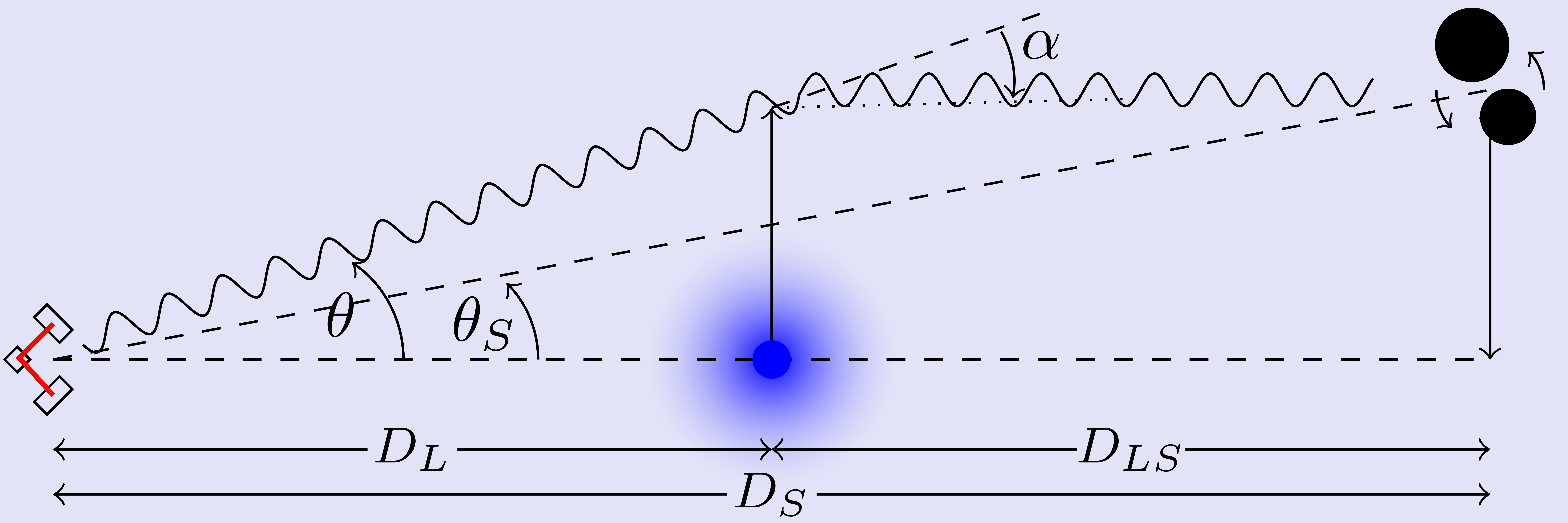
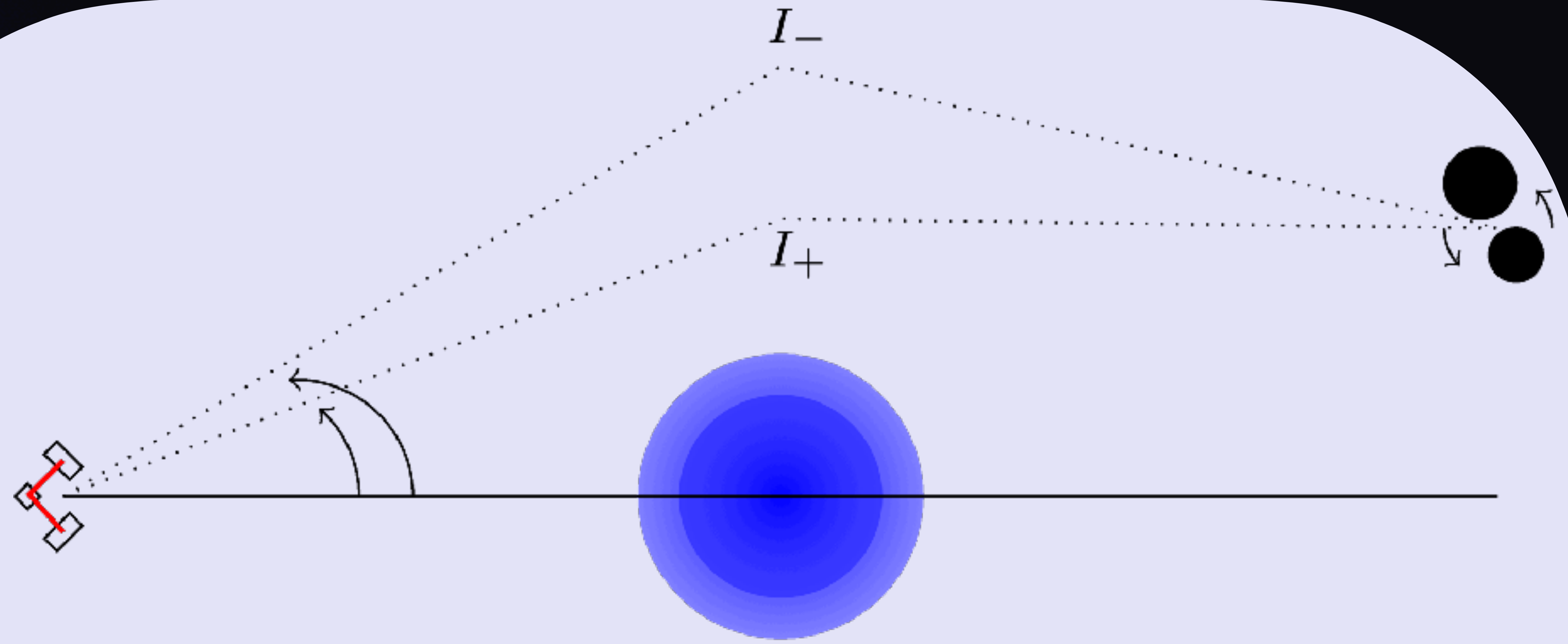
Gravitational Lensing of Gravitational Waves

Gravitational Lensing

EM: $10^3 < f < 10^{18}\ Hz$ | GW: $10^{-9} < f < 10^4\ Hz$



Geometrical-Optics vs Wave-Optics



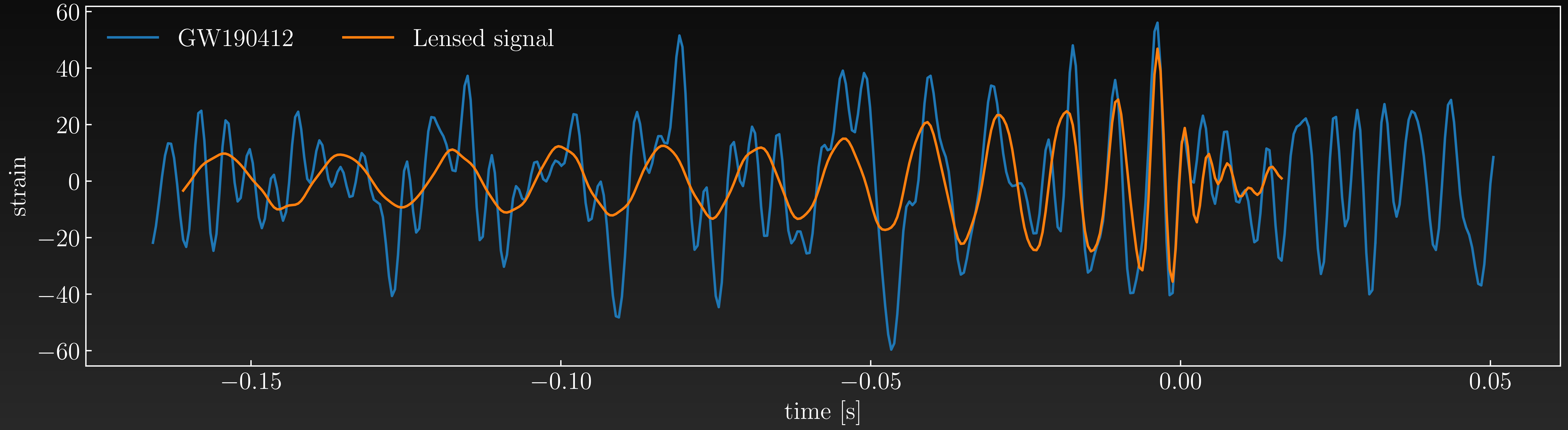
GO approximation breaks when

$$M_{3D,L} \leq 10^5 M_\odot \left[\frac{(1+z_L)f}{\text{Hz}} \right]^{-1}$$

$$f \cdot \Delta t \leq 1$$

Phase study of Lensed events

Unlensed vs lensed events



	Real parameter	Lensed parameter
$\mathscr{M}_{ob}\ [M_\odot]$	15.2	15.2
q	0.27	1
SNR	17.8 3 detectors	9 single detector

Lens

$$M_{L,ob} = 303\ M_\odot$$

$$y = \frac{\theta_S}{\theta_E} = 2$$

Unlensed vs lensed events

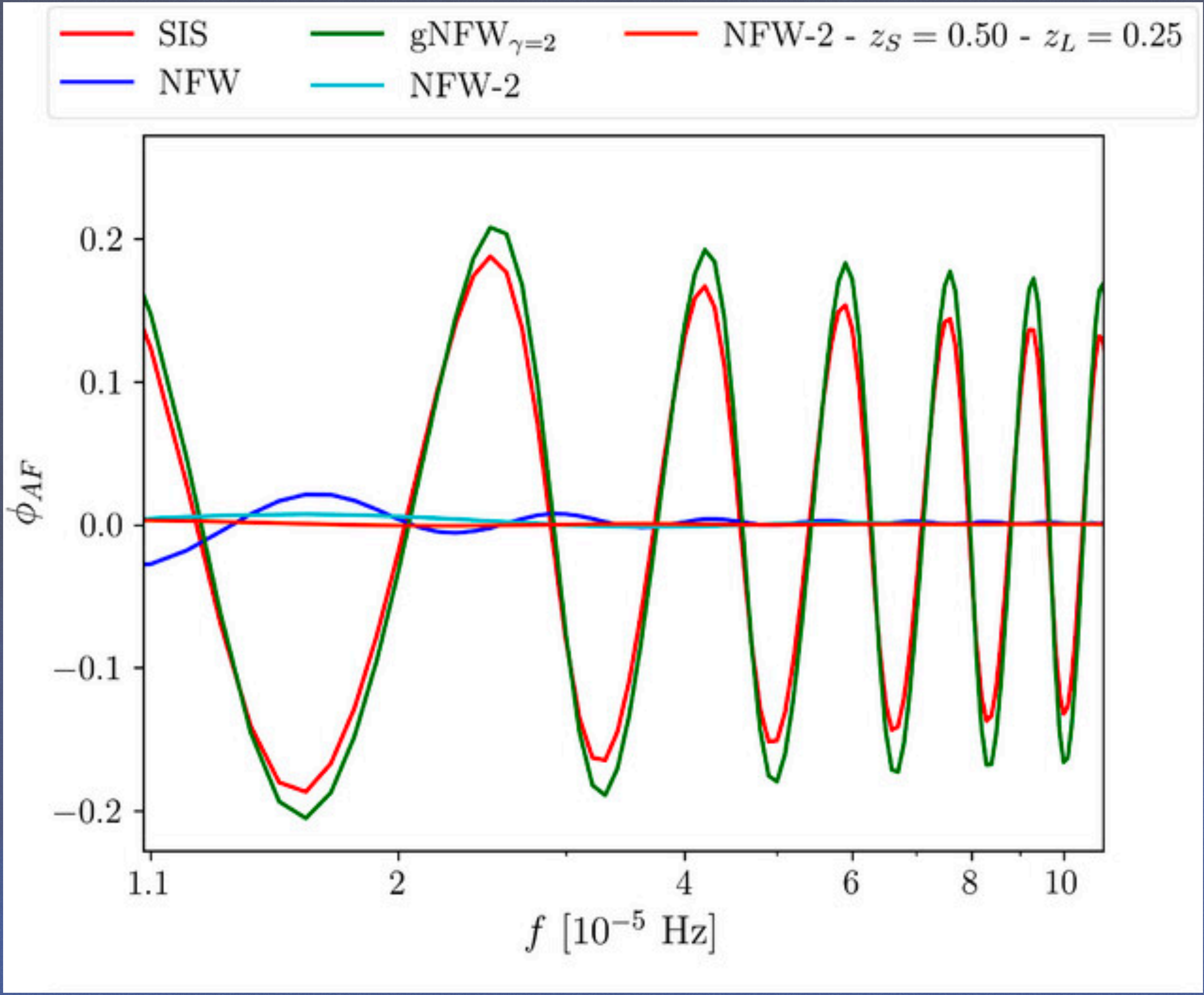
Gravitational Wave lensing

amplification factor

$$\hat{h}_L(f) = \hat{h}(f) \cdot F(f, y)$$

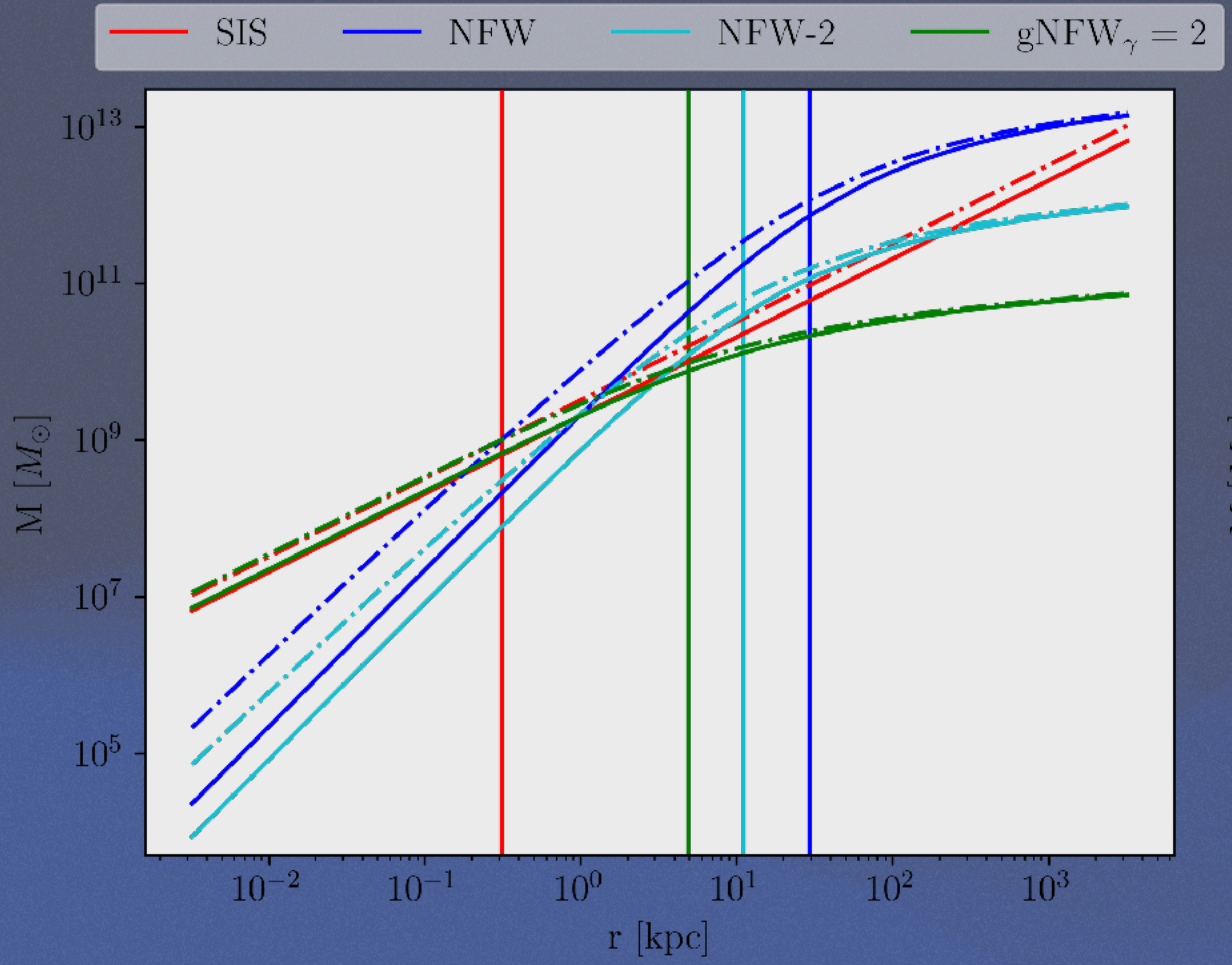
unlensed waveform
frequency domain

lensed waveform
frequency domain

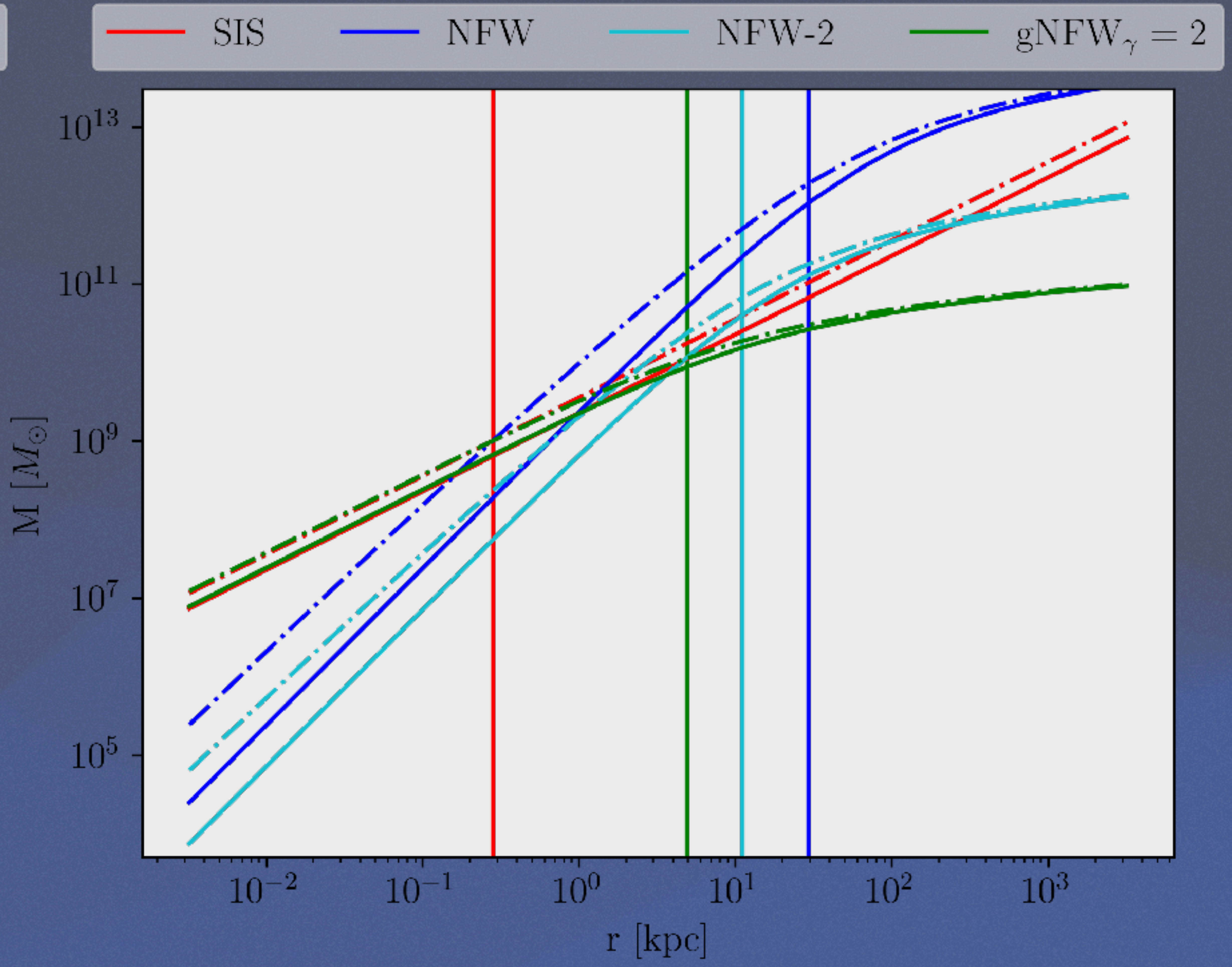


Lens mass profile

$z_L = 0.5$



$z_L = 0.15$



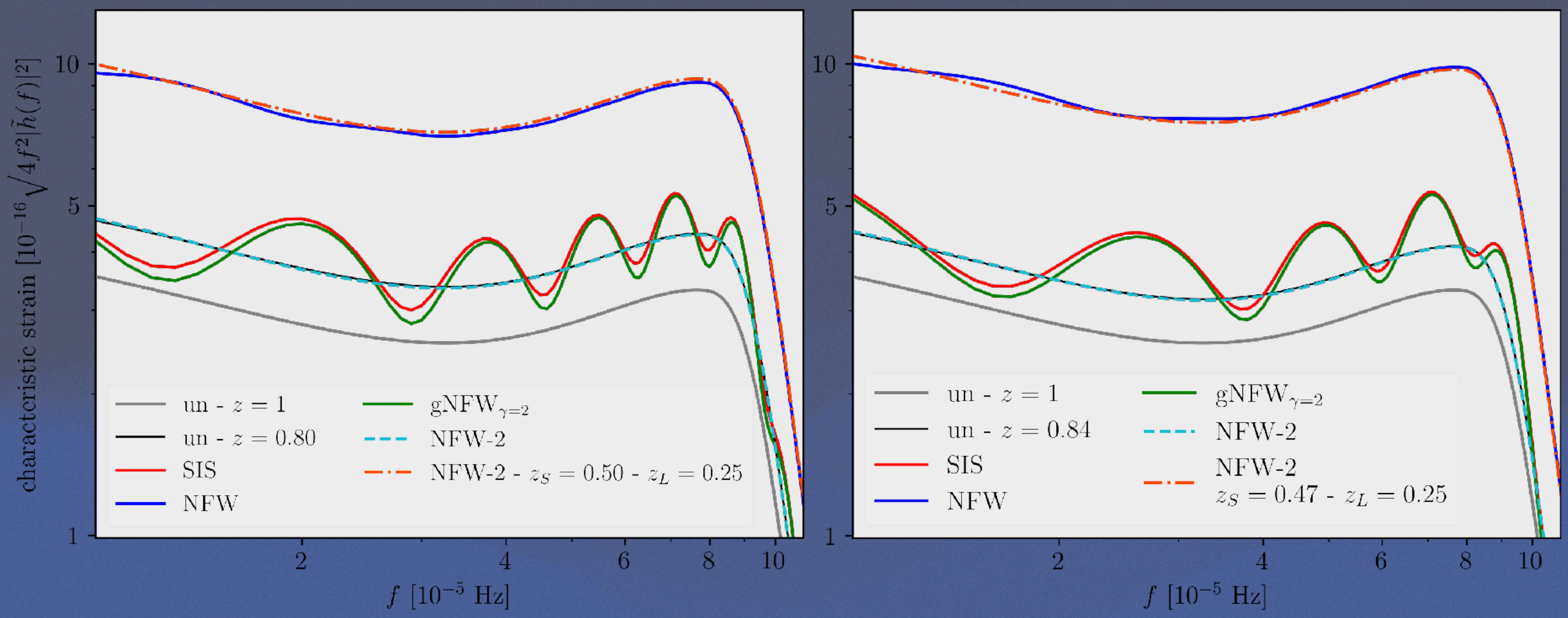
Lensed waveforms

$z_S = 1$

$M_s = 10^8\ M_\odot$

$M_L(r_c) = 10^9\ M_\odot$

$z_L = 0.5$ $\qquad$ $z_L = 0.15$



Lensed Waveforms

match study

$$\rho = \frac{\left(s \quad h_T\right)}{\sqrt{(h_T \quad h_T)}} \approx \frac{\left(h \quad h_T\right)}{\sqrt{(h_T \quad h_T)}}$$

- $s(t) = h(t) + n(t)$
- Inner product:

$$(a \quad b) = 2\,\mathrm{Re}\left[\int_0^\infty \frac{\tilde{a}(f)\cdot\tilde{b}^*(f) + \tilde{a}^*(f)\cdot\tilde{b}(f)}{S_n(f)} df\right]$$

  - $S_n(f)$ - (single-sided) power spectral density

$$\Delta\chi^2 \approx 2\rho_{opt}^2\left[1 - \frac{\rho}{\rho_{opt}}\right]$$

$$3\sigma \rightarrow \Delta\chi^2 \approx 11.8\ (14.2)$$

M. Maggiore,Gravitational Waves: Volume 1: Theory and Experiments,Gravitational Waves (OUP Oxford, 2008)

Lensed Waveforms phase study

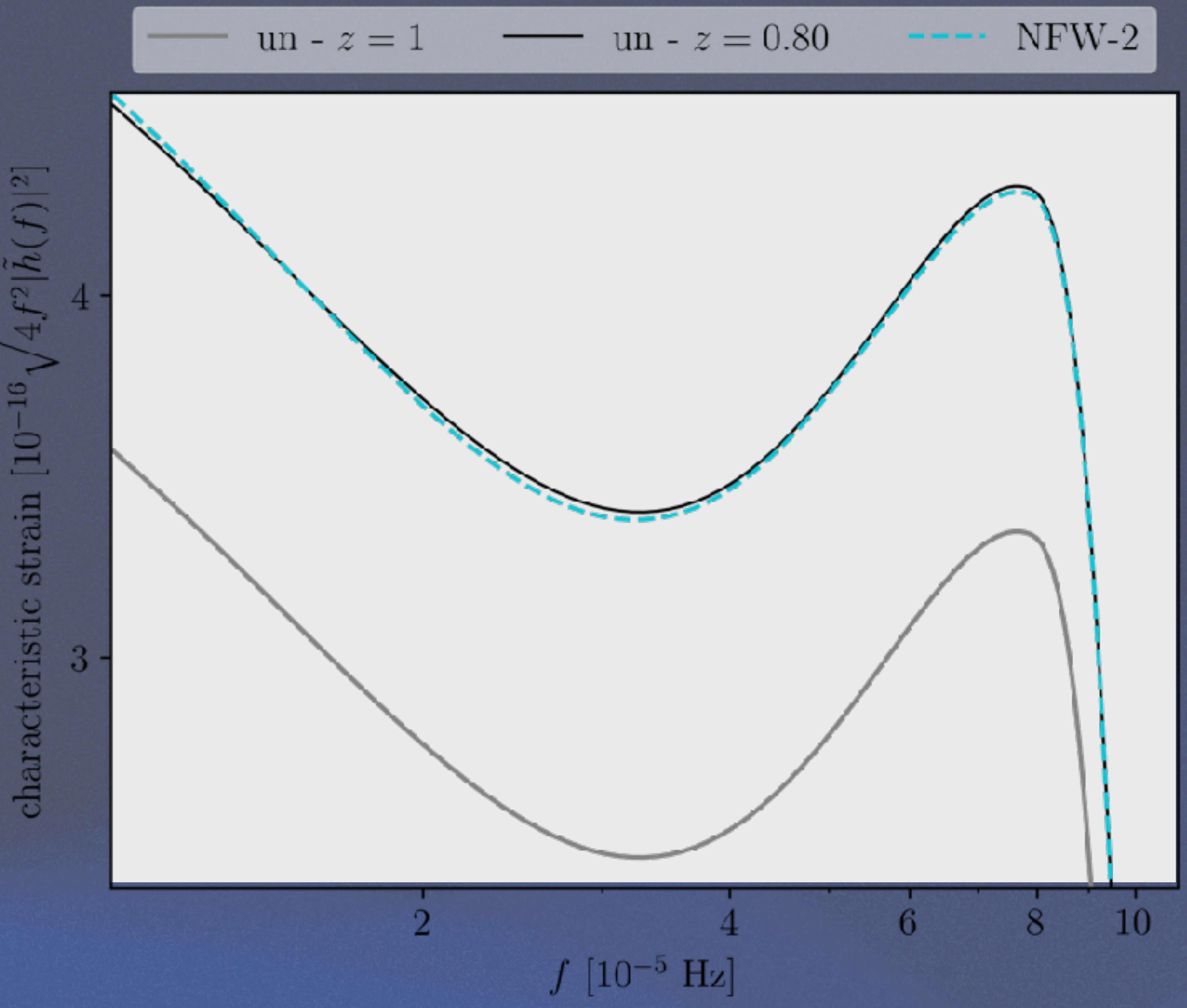
In a matched filtering analysis, the phase of the waveform can be measured with an accuracy corresponding to the inverse SNR

$$\sigma_\phi \approx \rho^{-1} \text{ rad}$$

C. Cutler, E. E. Flanagan, Phys. Rev. 1994, D49, 2658.

Unlensed vs lensed

Lensed waveforms



$\rho \approx 220$ SNR of the signal

$\frac{\rho}{\rho_{opt}} = 1 - 4 \cdot 10^{-7}$ lensed / unlensed

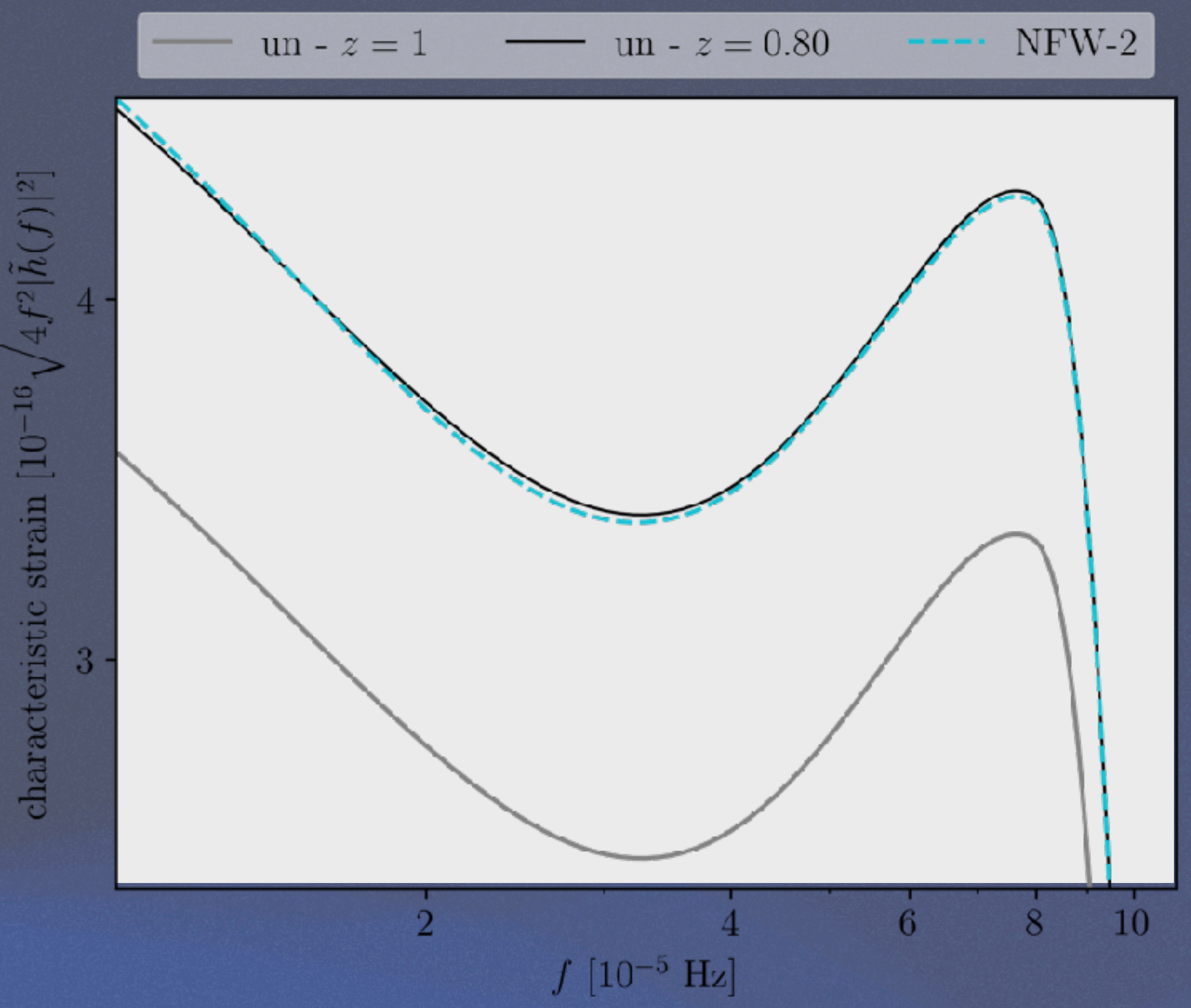
$\Delta\chi^2 \approx 14.2$ 3 free parameters

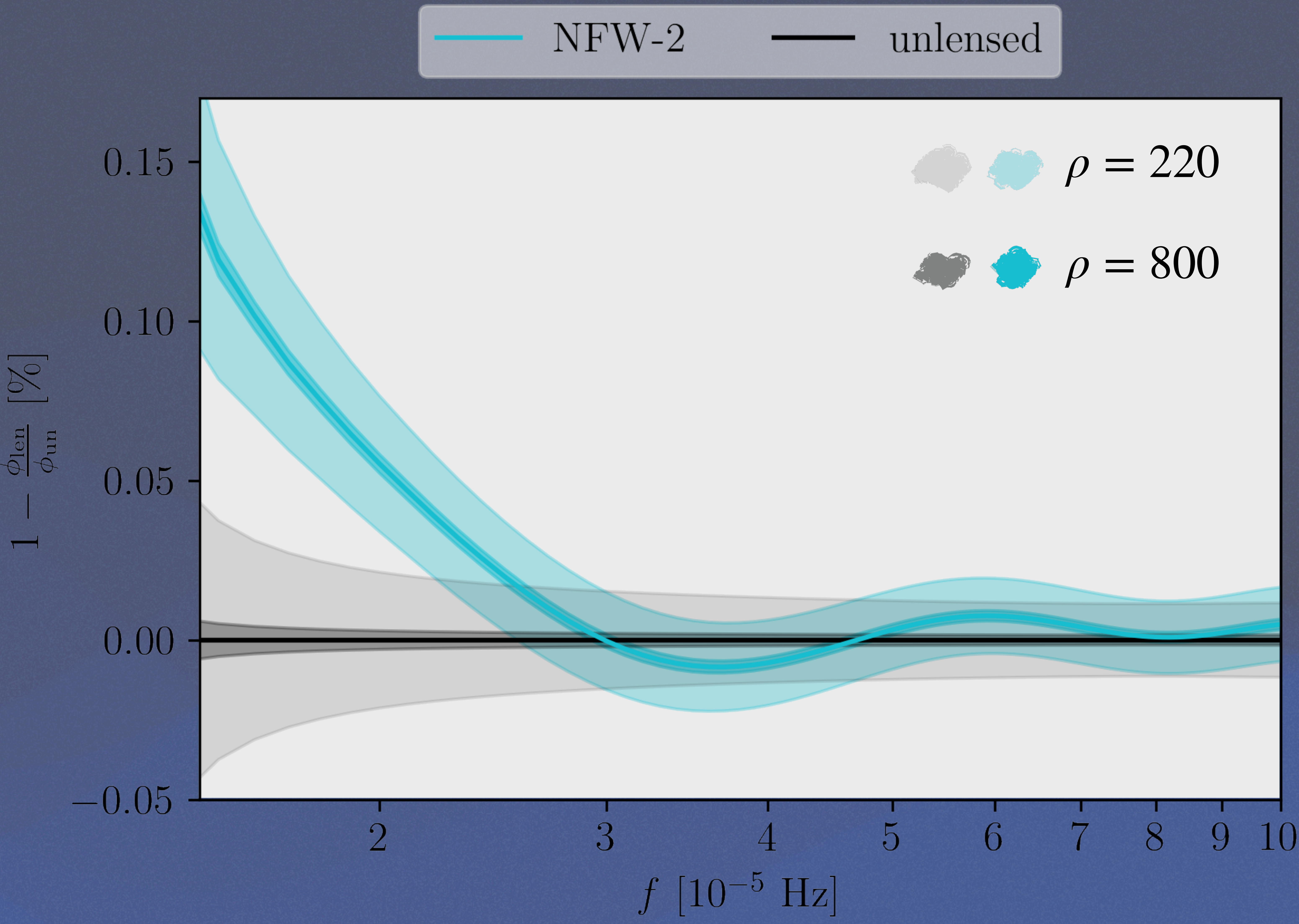
$\frac{\rho}{\rho_{opt}} = 1 - 1.5 \cdot 10^{-4}$ 3σ threshold

We would need $\rho \approx 4000$

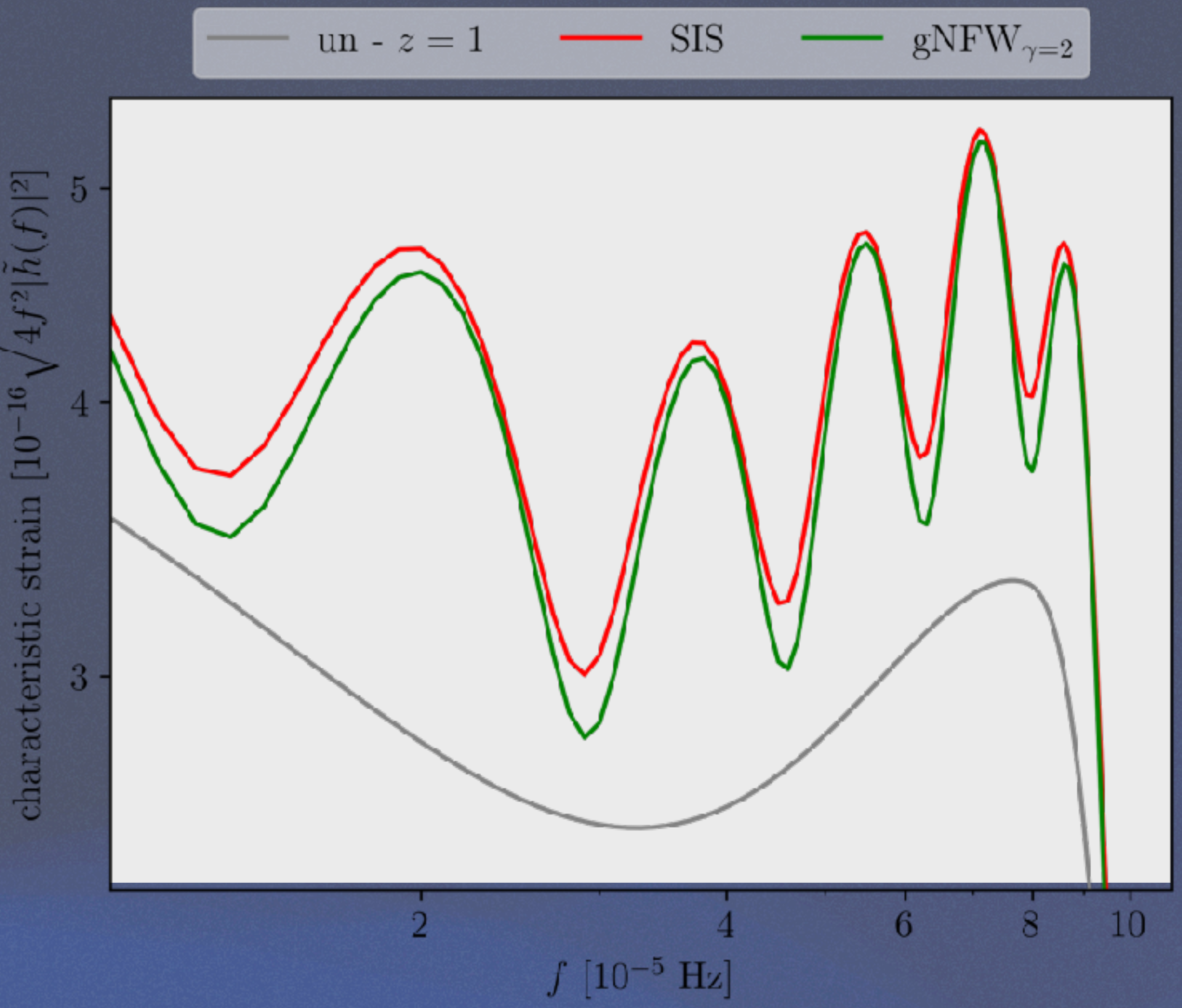
Unlensed vs lensed

Lensed waveforms





Constraining lens models



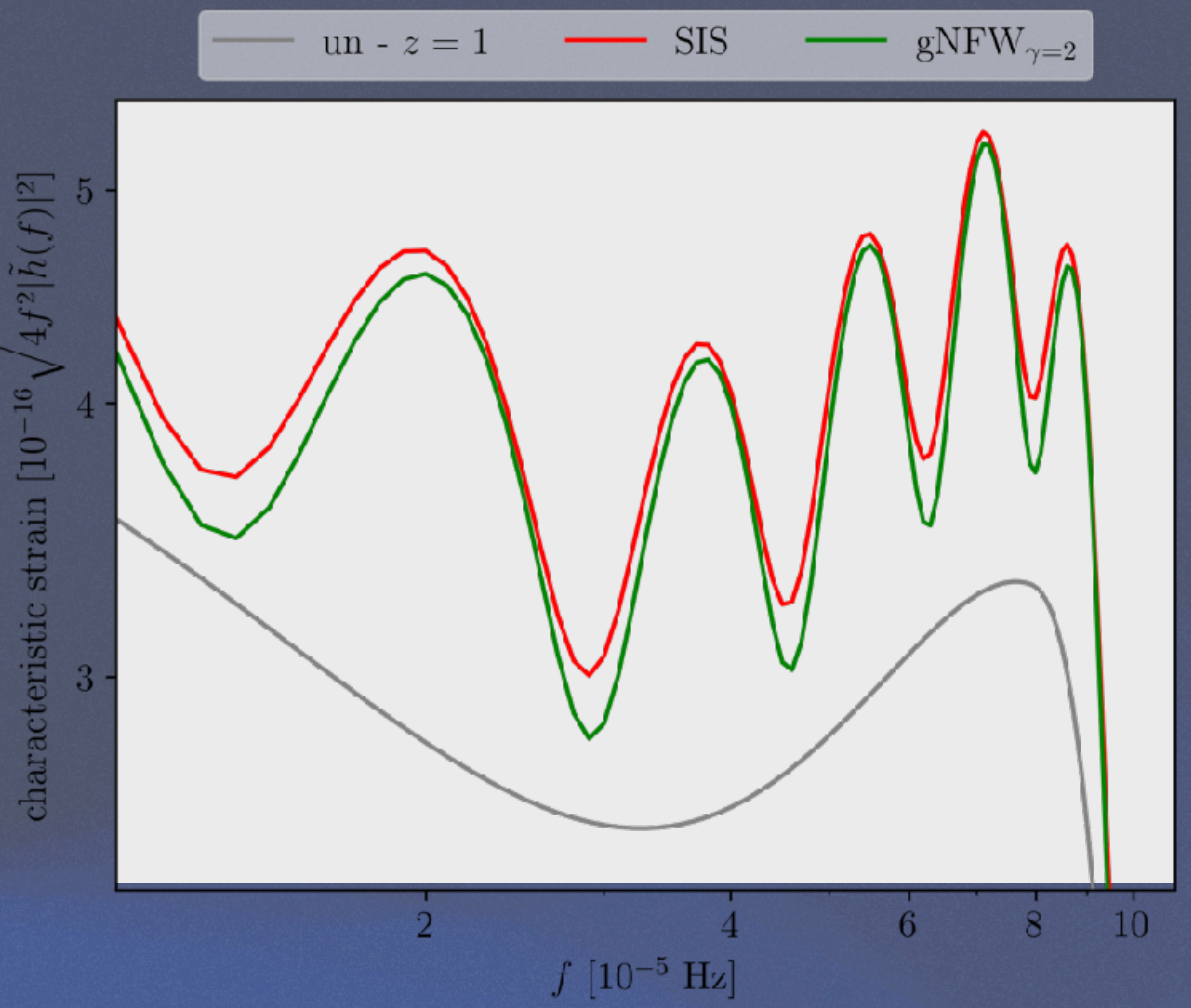
$\rho \approx 100$ SNR of the signal

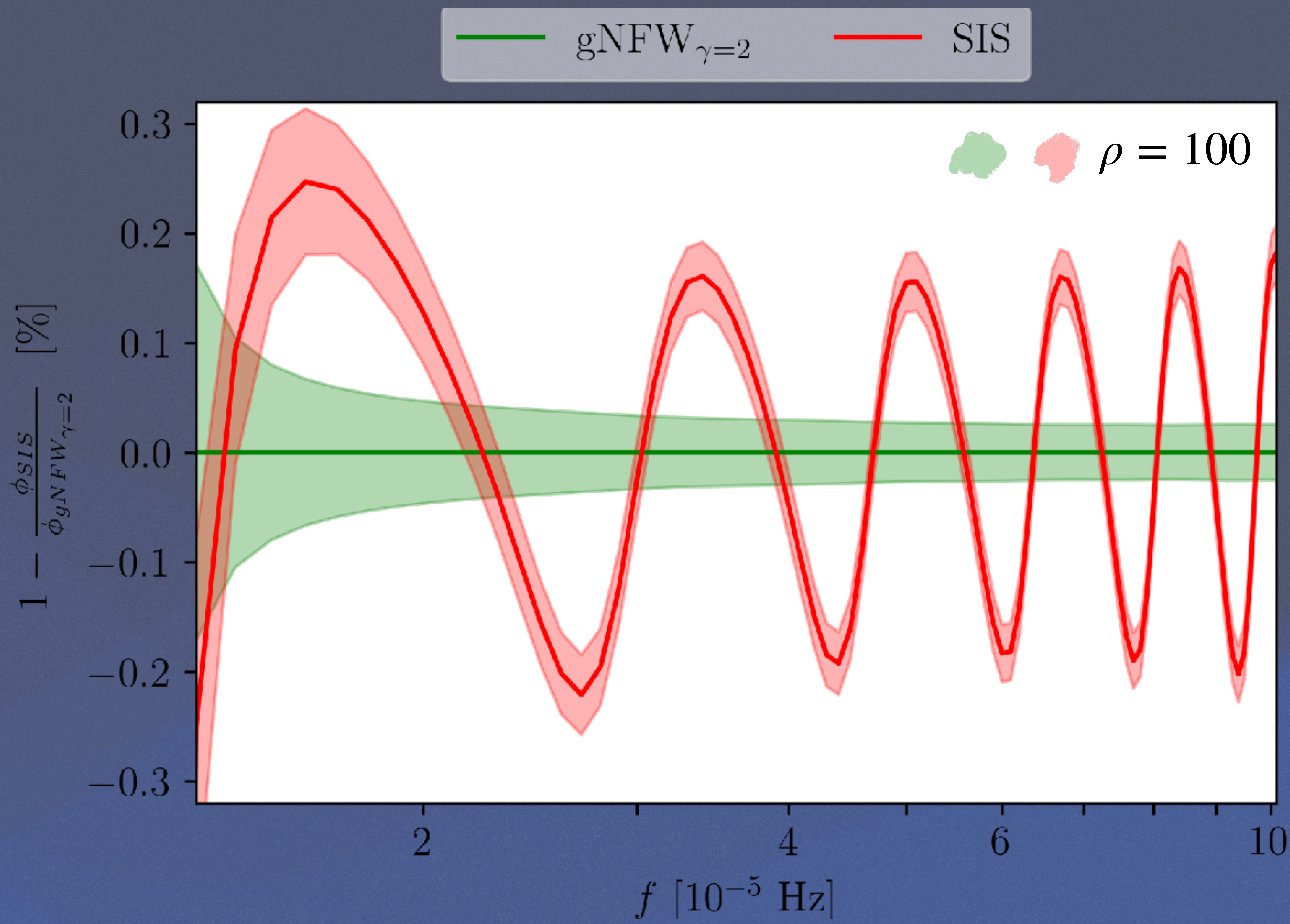
$\frac{\rho}{\rho_{opt}} = 0.9869$ SIS / $\mathrm{gNFW}_{\gamma=2}$

$\Delta\chi^2 \approx 11.8$ 2 free parameters

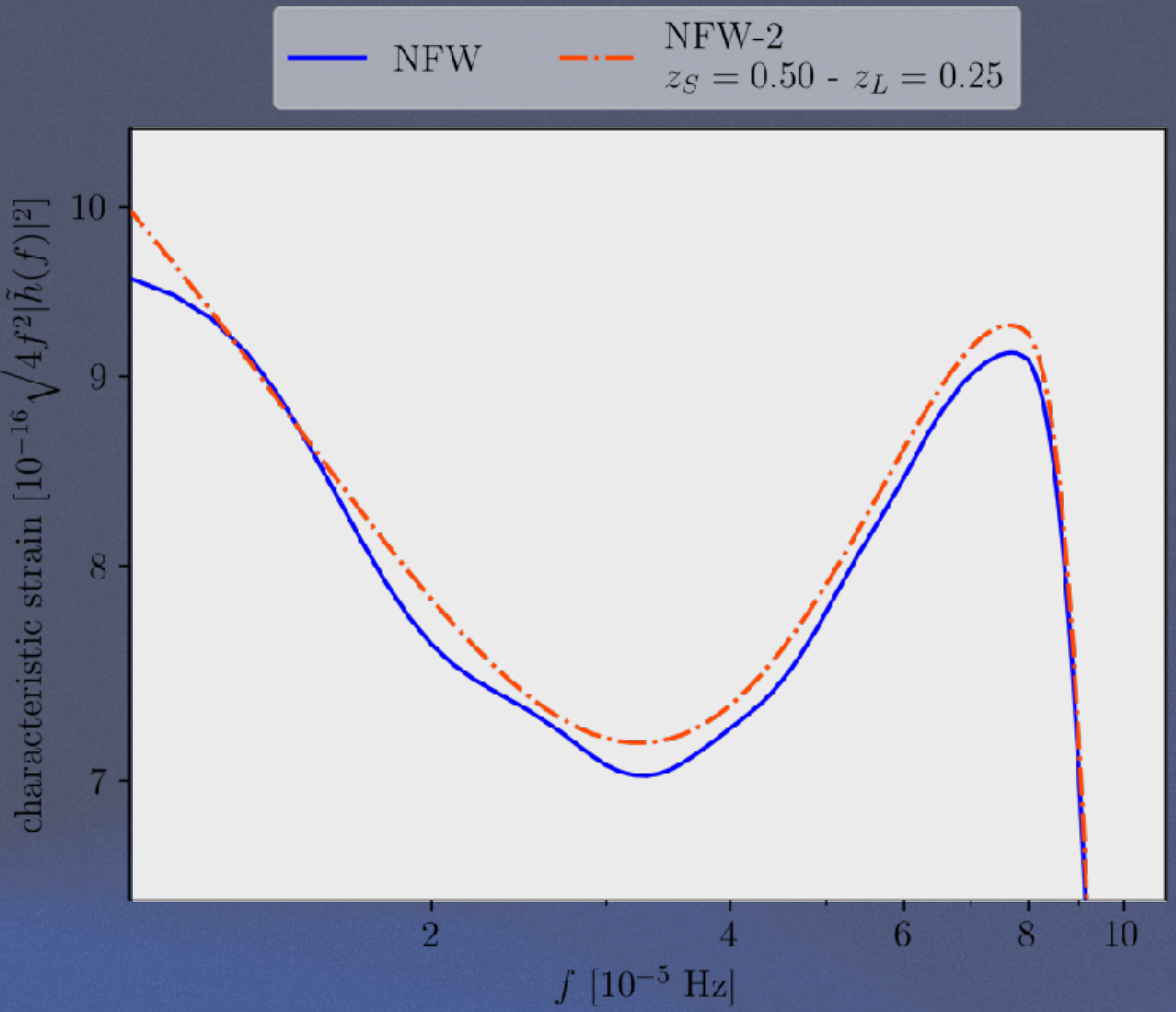
$\frac{\rho}{\rho_{opt}} = 0.9994$ 3σ threshold

Constraining lens models





Constraining lens models



$\rho \approx 220$ SNR of the signal

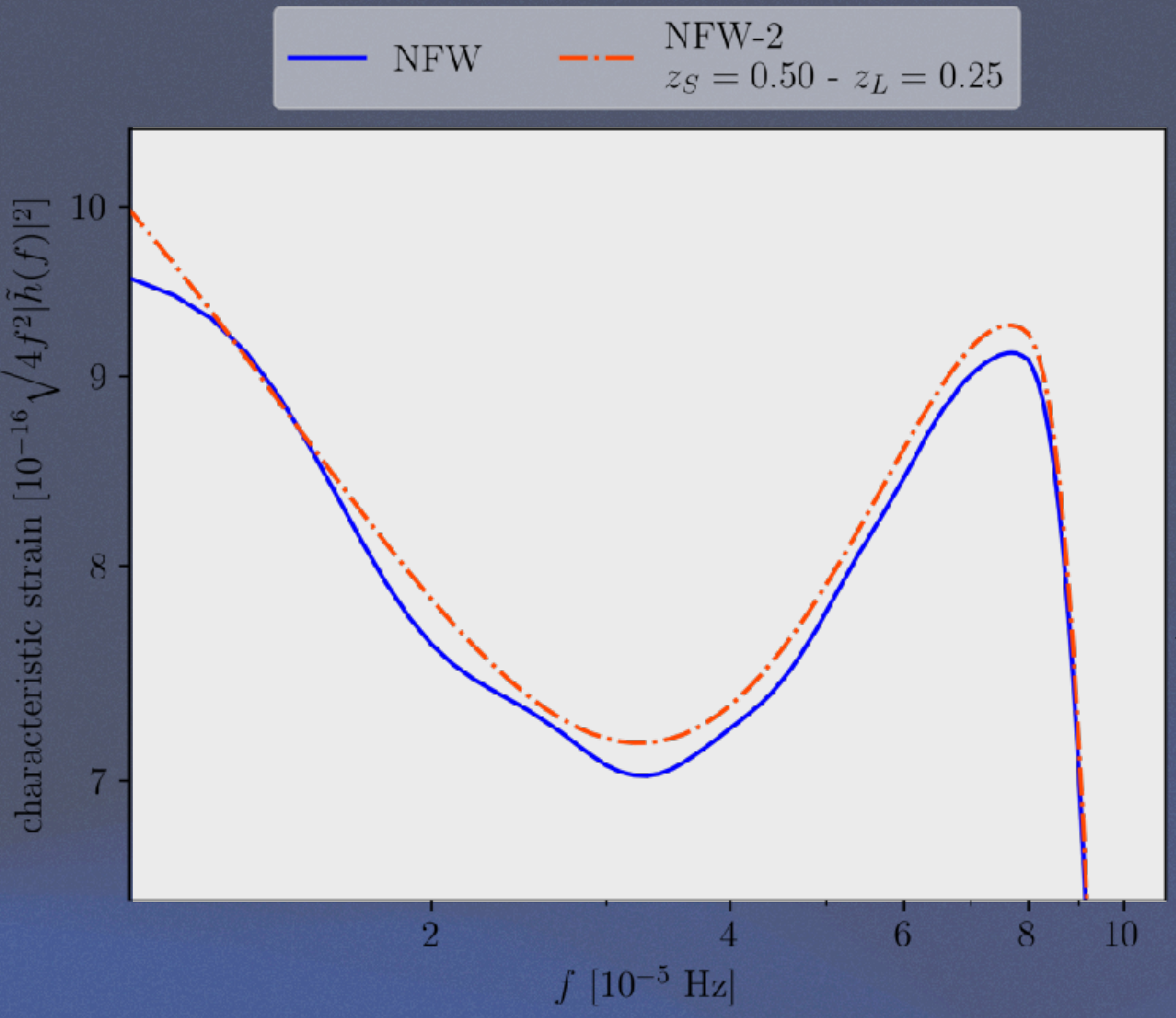
$\frac{\rho}{\rho_{opt}} = 1 - 1.4 \cdot 10^{-6}$ NFW / NFW-2

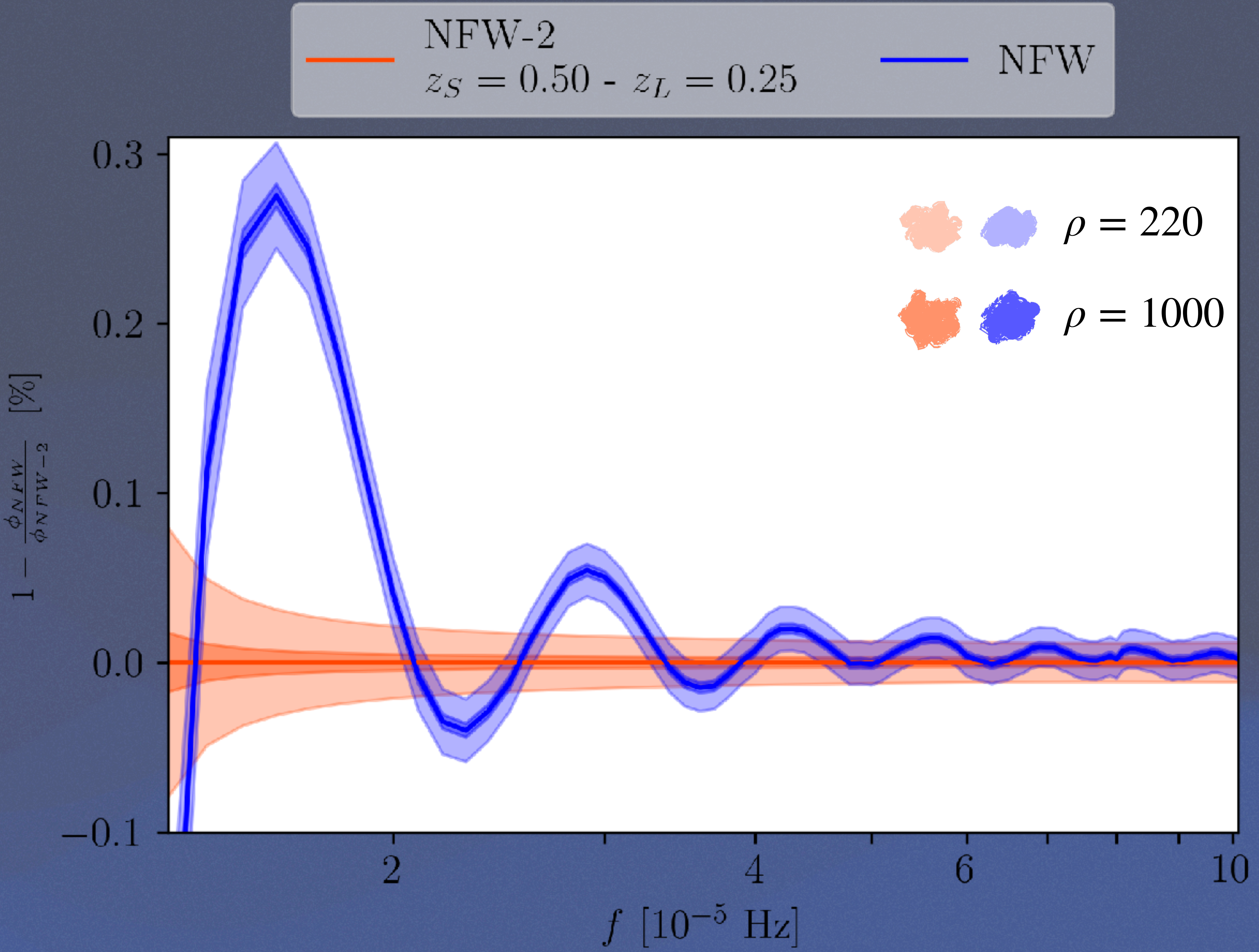
$\Delta\chi^2 \approx 14.2$ 3 free parameters

$\frac{\rho}{\rho_{opt}} = 1 - 1.4 \cdot 10^{-4}$ 3σ threshold

We would need $\rho \approx 2200$

Constraining lens models





Conclusions 1/2

- Lensed events can be misinterpreted by unlensed one
- Studying the phase of the signal is more effective than matched filtering
- We can differentiate between lens models
- Differentiating between models is useful to study dark matter/dark energy content

Mass-Sheet Degeneracy

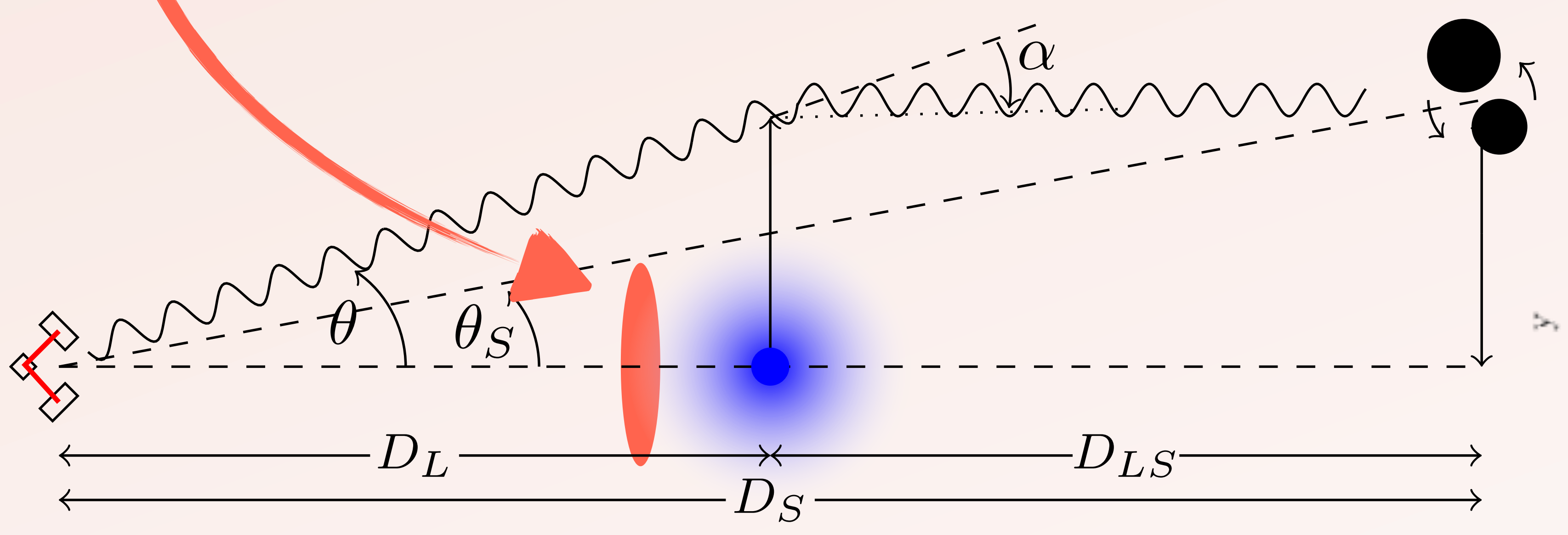
Mass-Sheet Degeneracy in Gravitational-Waves microlensing

1. **MSD transformation** $\rightarrow \lambda$

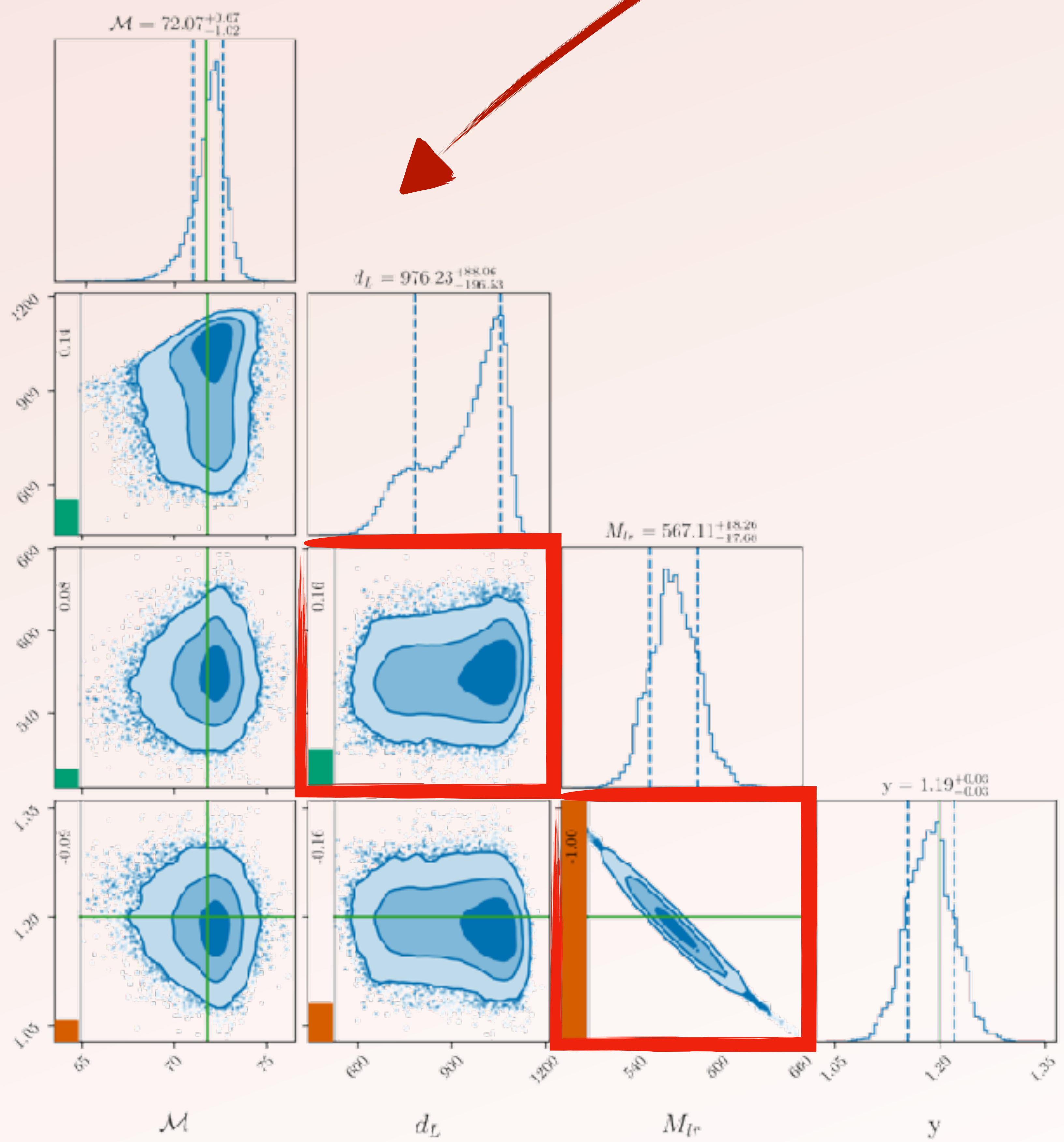
Scalings of lens mass &
Scaling of angles

Effectively adding (subtracting) mass to the lens w/ no visible angular change

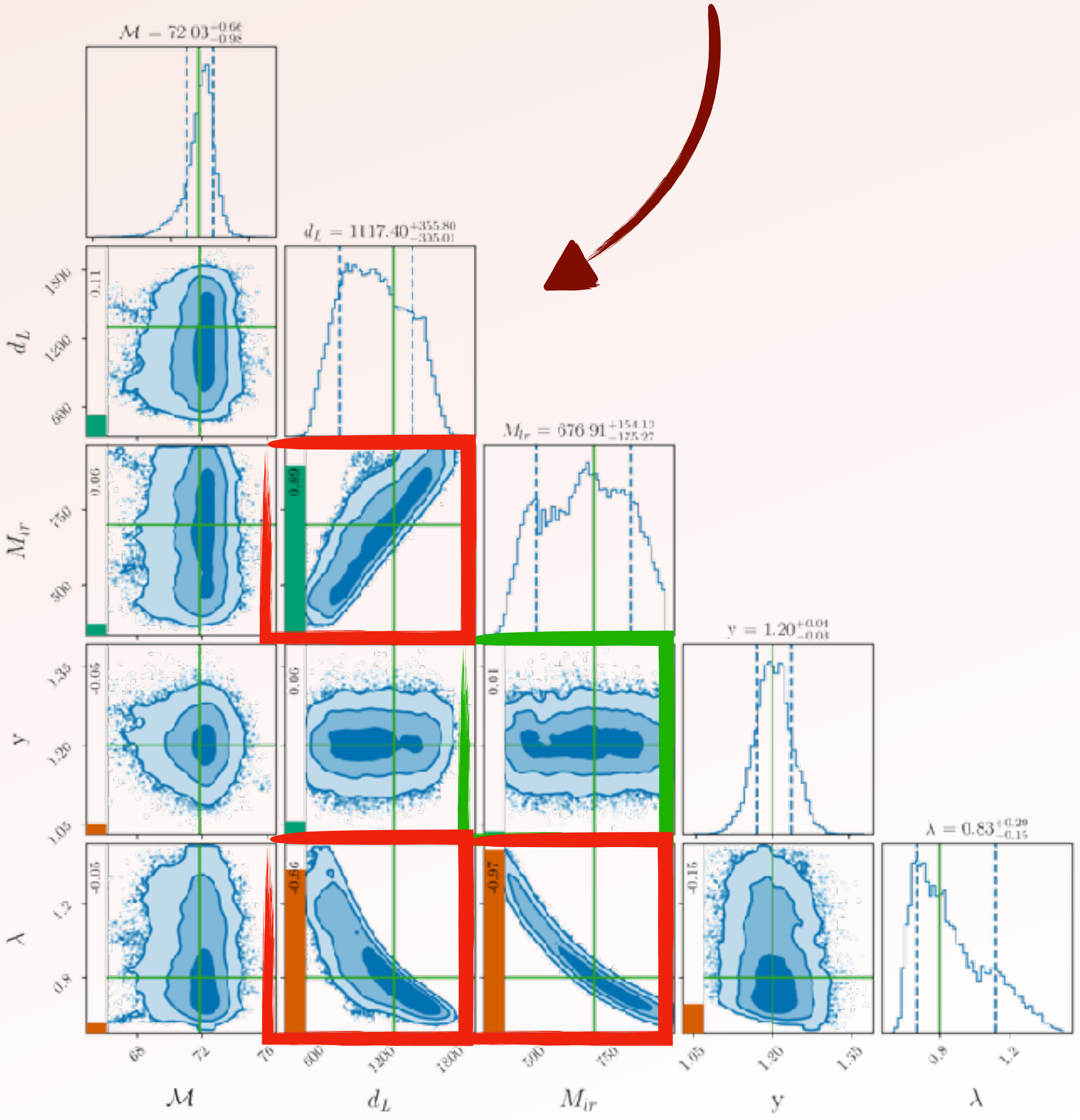
Biased estimations of lens parameters!



2. Simulate a BBH lensed event with $\lambda \neq 1$



3. Study the event through 2 PE analysis:
   a. MSD parameter fixed to $\lambda = 1$
   b. MSD parameter free



Mass-Sheet Degeneracy

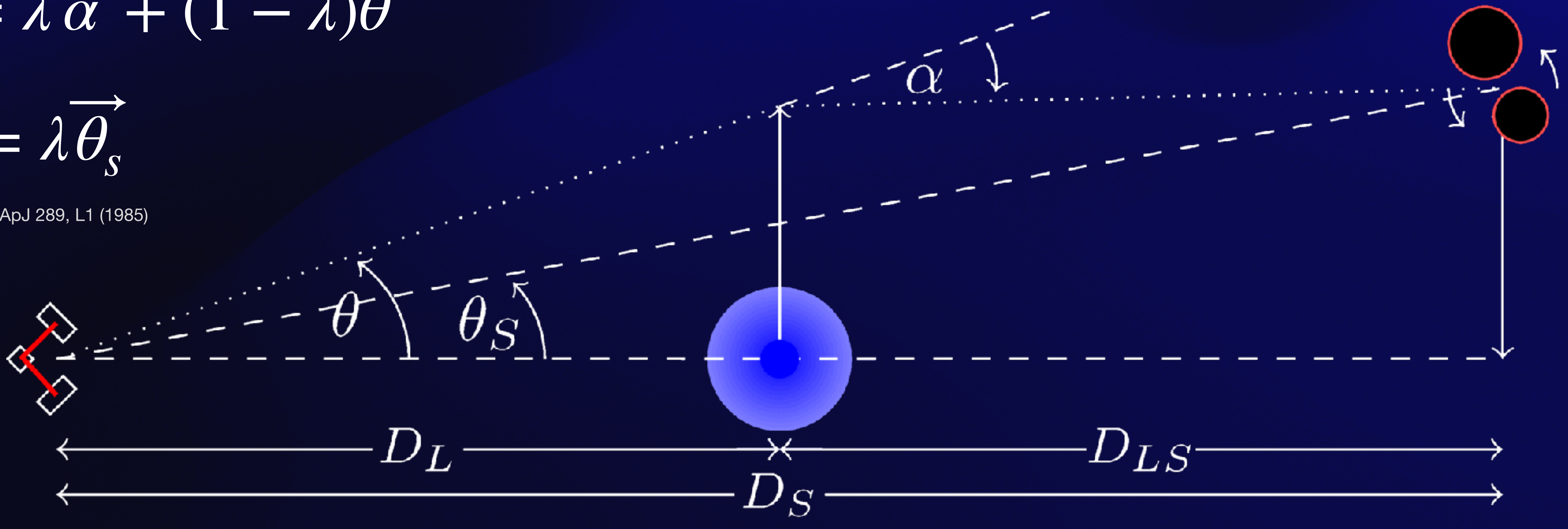
- Scalings of lens mass:
  - $\kappa \rightarrow \kappa_\lambda = \lambda\kappa + (1 - \lambda)$

$\kappa = \Sigma/\Sigma_{cr}$

Σ - surface mass density

- Scaling angles:
  - $\overrightarrow{\alpha} \rightarrow \overrightarrow{\alpha}_\lambda = \lambda\overrightarrow{\alpha} + (1 - \lambda)\vec{\theta}$
  - $\overrightarrow{\theta_s} \rightarrow \vec{\theta}_{s,\,\lambda} = \lambda\overrightarrow{\theta_s}$

E. E. Falco, M. V. Gorenstein, and I. I. Shapiro, ApJ 289, L1 (1985)



Mass-Sheet Degeneracy

- Scalings of lens mass:
  - $\kappa \rightarrow \kappa_\lambda = \lambda\kappa + (1 - \lambda)$
- Scaling angles:
  - $\overrightarrow{\alpha} \rightarrow \overrightarrow{\alpha}_\lambda = \lambda\overrightarrow{\alpha} + (1 - \lambda)\vec{\theta}$
  - $\overrightarrow{\theta_s} \rightarrow \vec{\theta}_{s,\,\lambda} = \lambda\overrightarrow{\theta_s}$

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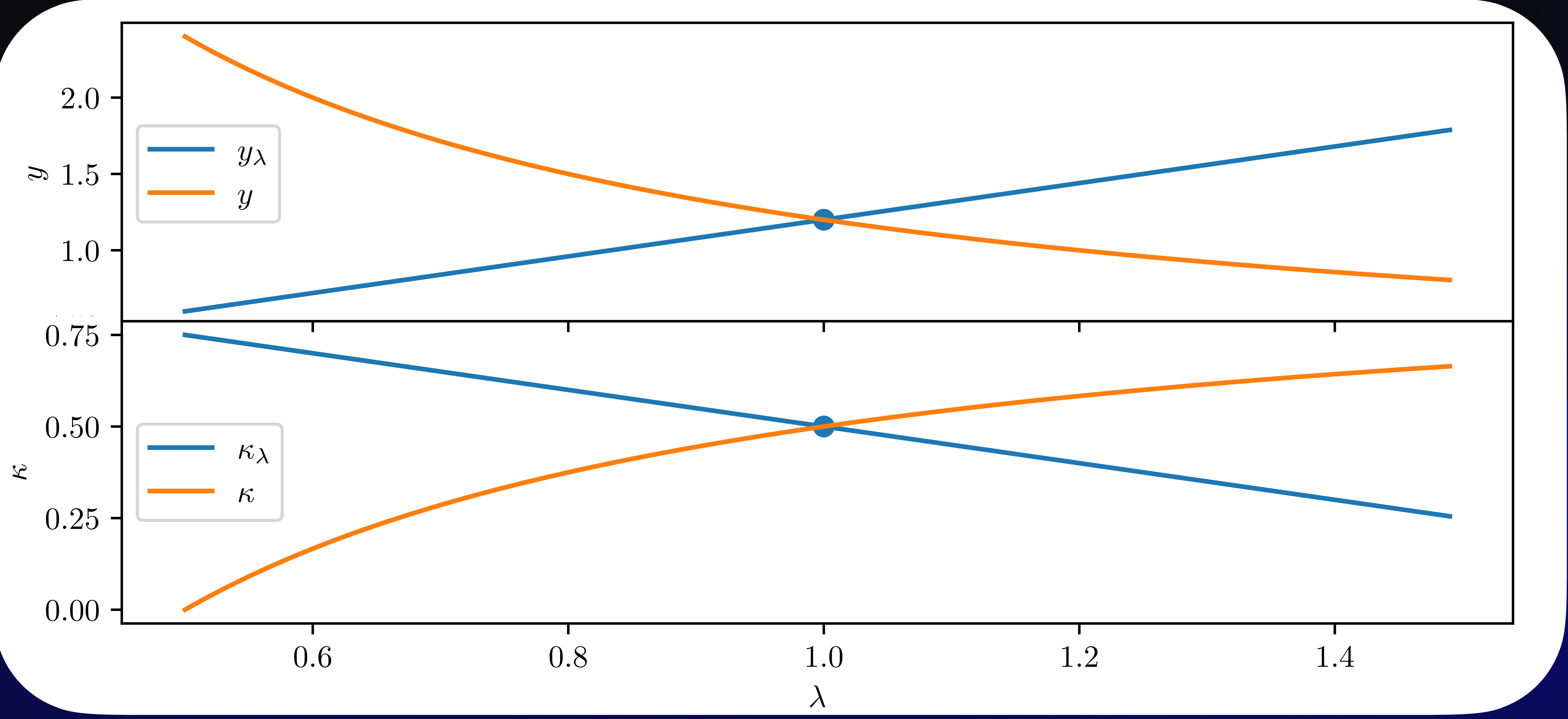


Re-parametrisation of lens parameter:

$$\theta_S \rightarrow y = \frac{\theta_S}{\theta_E}$$

where

θ_E: Einstein radius

Mass-Sheet Degeneracy

Why a problem?

- Observables are preserved!
- Problems: e.g. biased estimations of mass lens
- Biased estimation of cosmological parameter, e.g. H_0

Can we solve it?

- EM geometrical optics regime: multiple images; independent mass estimation of the lens (e.g. dynamics)
- EM wave optics regime: multiple lenses
- **In GW lensing: 1 image and 1 lens can break MSD!**

Gravitational Lensing of Gravitational Waves

amplification
factor

$$\hat{h}_L(f) = \hat{h}(f) \cdot F(f, y)$$

lensed waveform
frequency domain

unlensed waveform
frequency domain

Gravitational Lensing of Gravitational Waves

λ dependent amplification factor

$$\hat{h}_L(f) = \hat{h}(f) \cdot F(f, y, \lambda)$$

lensed waveform
frequency domain

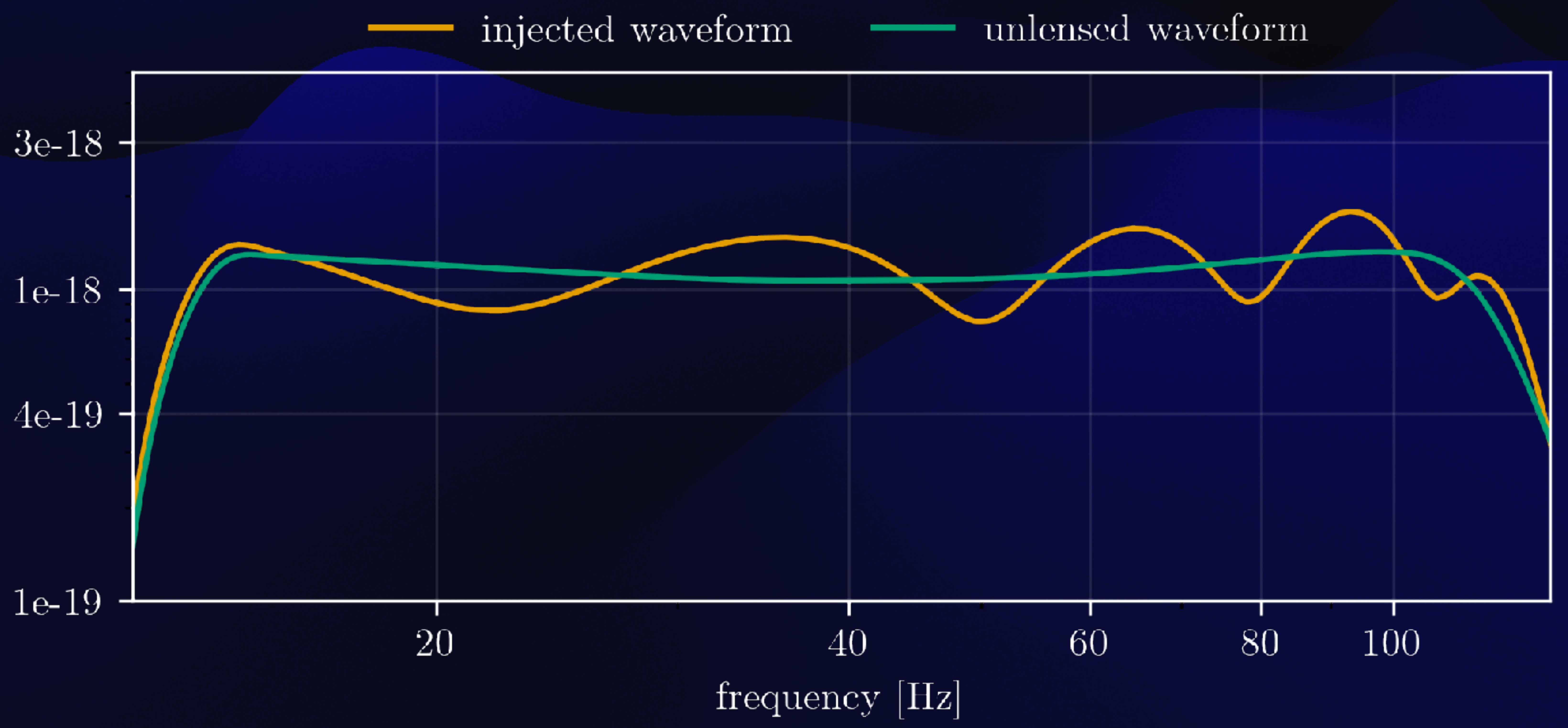
unlensed waveform
frequency domain

Mass-Sheet Degeneracy

PE analysis

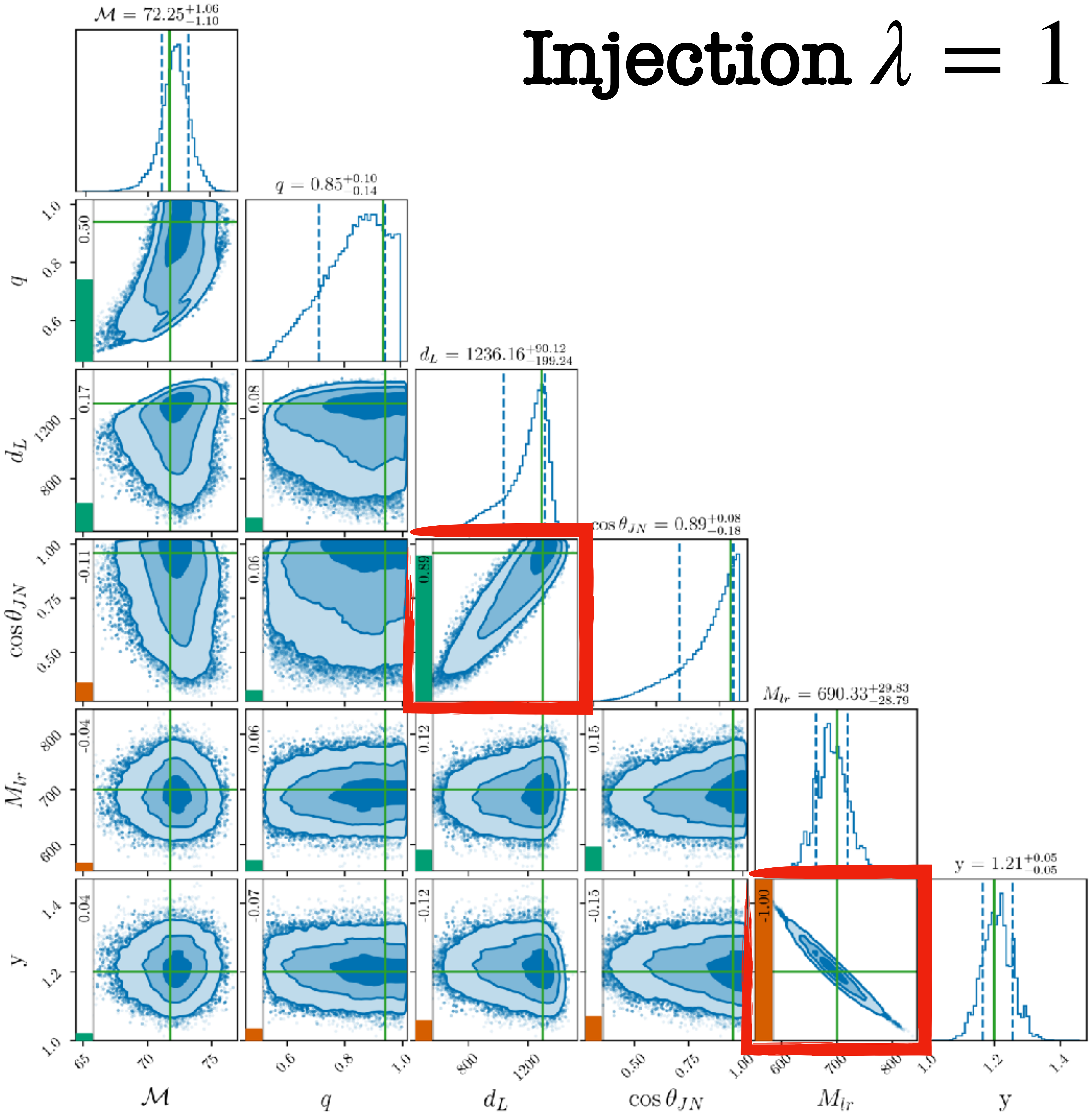
Mass-Sheet Degeneracy

Injection $\lambda = 1$



Parameter	Value
$\mathscr{M}$	71.78
q	0.94
d_L [Mpc]	1300
$\cos\theta_{JN}$	0.95
$M_{l,r}$ [$M_\odot$]	700
y	1.2
λ	1
detectors	H1,L1,V1
optimal SNR	78
wf approx	IMRPhenomXP

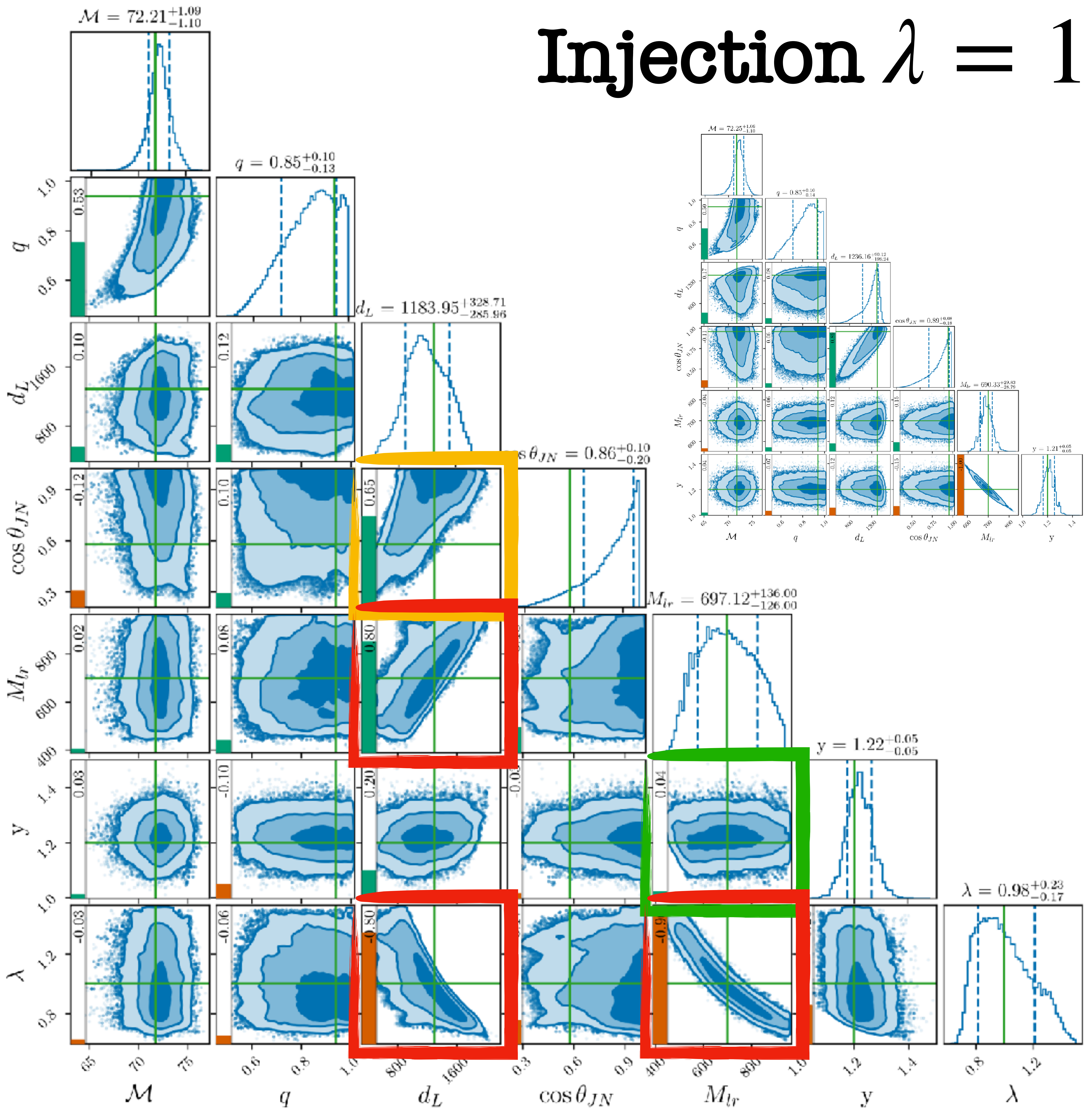
Injection $\lambda = 1$



PE without λ

- High correlation $M_{l,r} \; - \; y$
- High correlation $d_L \; - \; \theta_{JN}$

Injection $\lambda = 1$

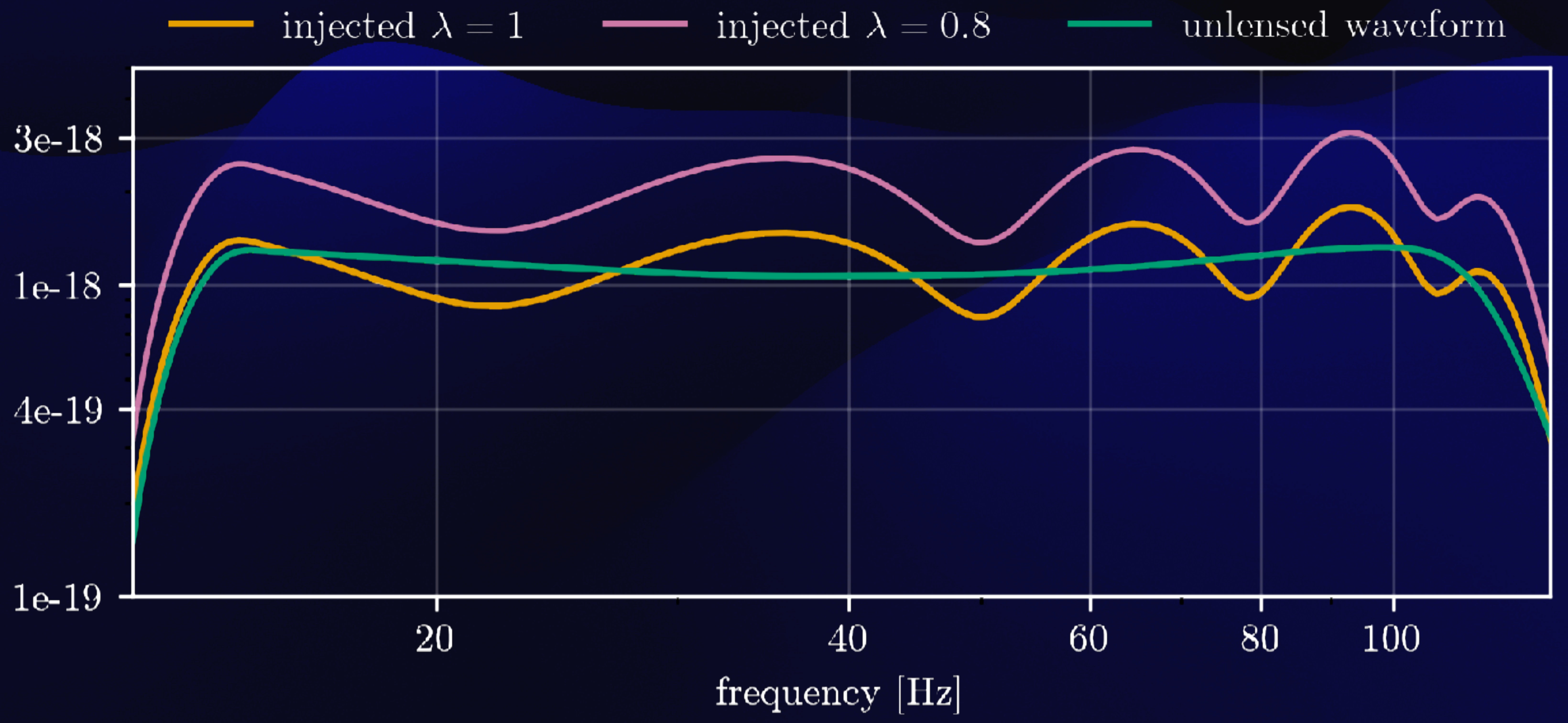


PE with λ

- NO CORR $M_{l,r}$ – y
- correlation to $M_{l,r}$ – λ
- luminosity distance d_L
  - smaller corr w/ θ_{JN}
  - high corr w/ $M_{l,r}$
  - high corr w/ λ

Mass-Sheet Degeneracy

Injection $\lambda = 0.8$



Parameter	Value
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q	0.94
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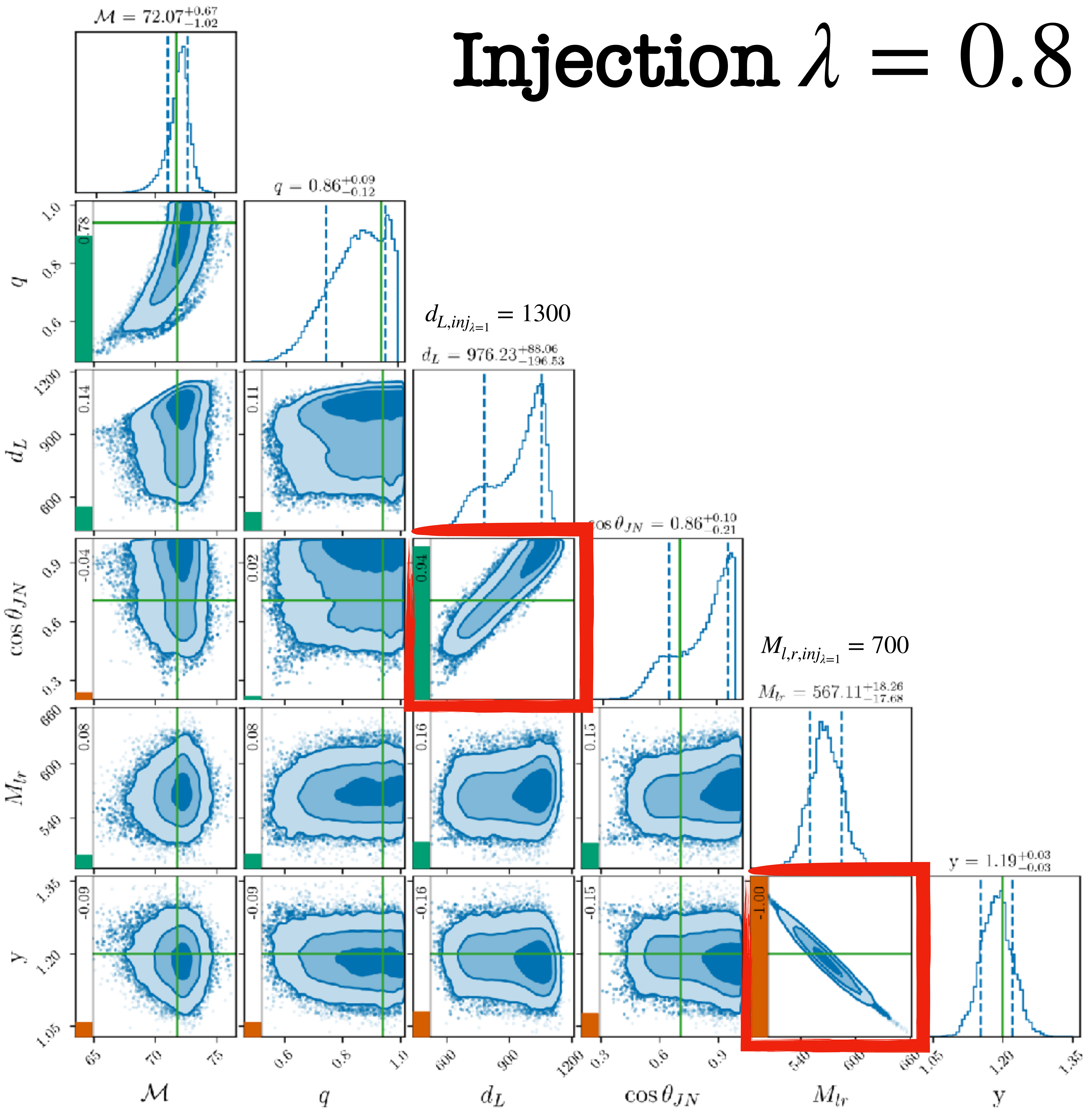
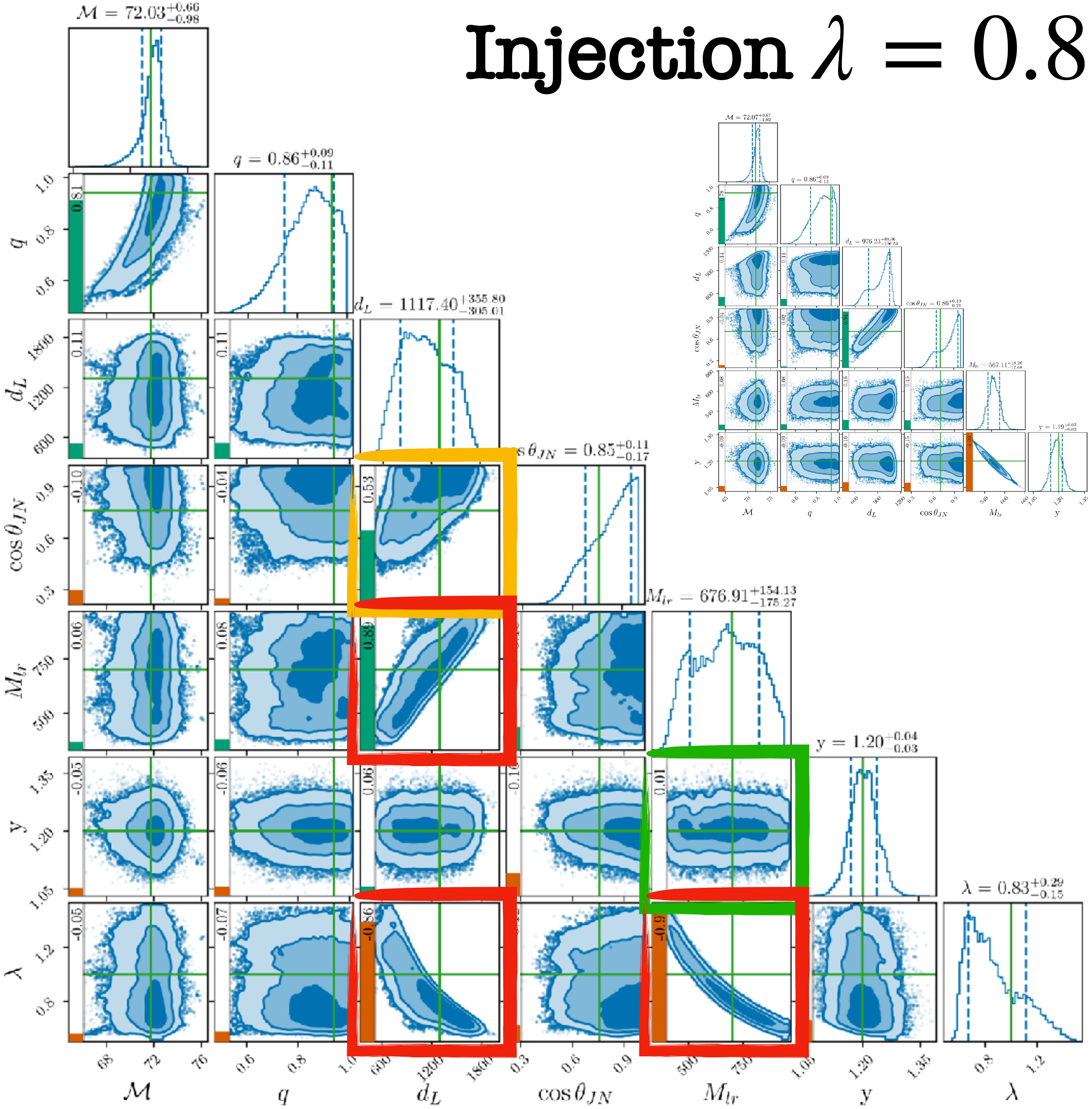
Injection $\lambda = 0.8$



PE without λ

- High correlation $M_{l,r}$ – y
- High correlation d_L – θ_{JN}
- Value of y OK
- Value of $M_{l,r}$ changes
  - absorbs $\lambda = 0.8$, not as expected
  - because of d_L and y

Injection $\lambda = 0.8$



PE with λ

- **All parameters retrieved correctly**
- NO CORR $M_{l,r}$ – y
- correlation to $M_{l,r}$ – λ
- luminosity distance d_L
  - smaller corr w/ θ_{JN}
  - high corr w/ $M_{l,r}$
  - high corr w/ λ

Mass-Sheet Degeneracy

Real(istic) events

Parameter	Injection	Injected GW200208	GW200208
$\mathscr{M}$	71.78	38.97	38.90
q	0.94	0.76	0.77
d_L [Mpc]	1300	2294.6	2770
$\cos\theta_{JN}$	0.95	-0.83	-0.83
$M_{l,r}$ $[M_\odot]$	700	1693.47	1900
y	1.2	1.46	1.5
λ	1	———	———
detectors	H1,L1,V1	H1,L1,V1	H1,L1,V1
optimal SNR	78	19.12	9.91

Injected GW200208

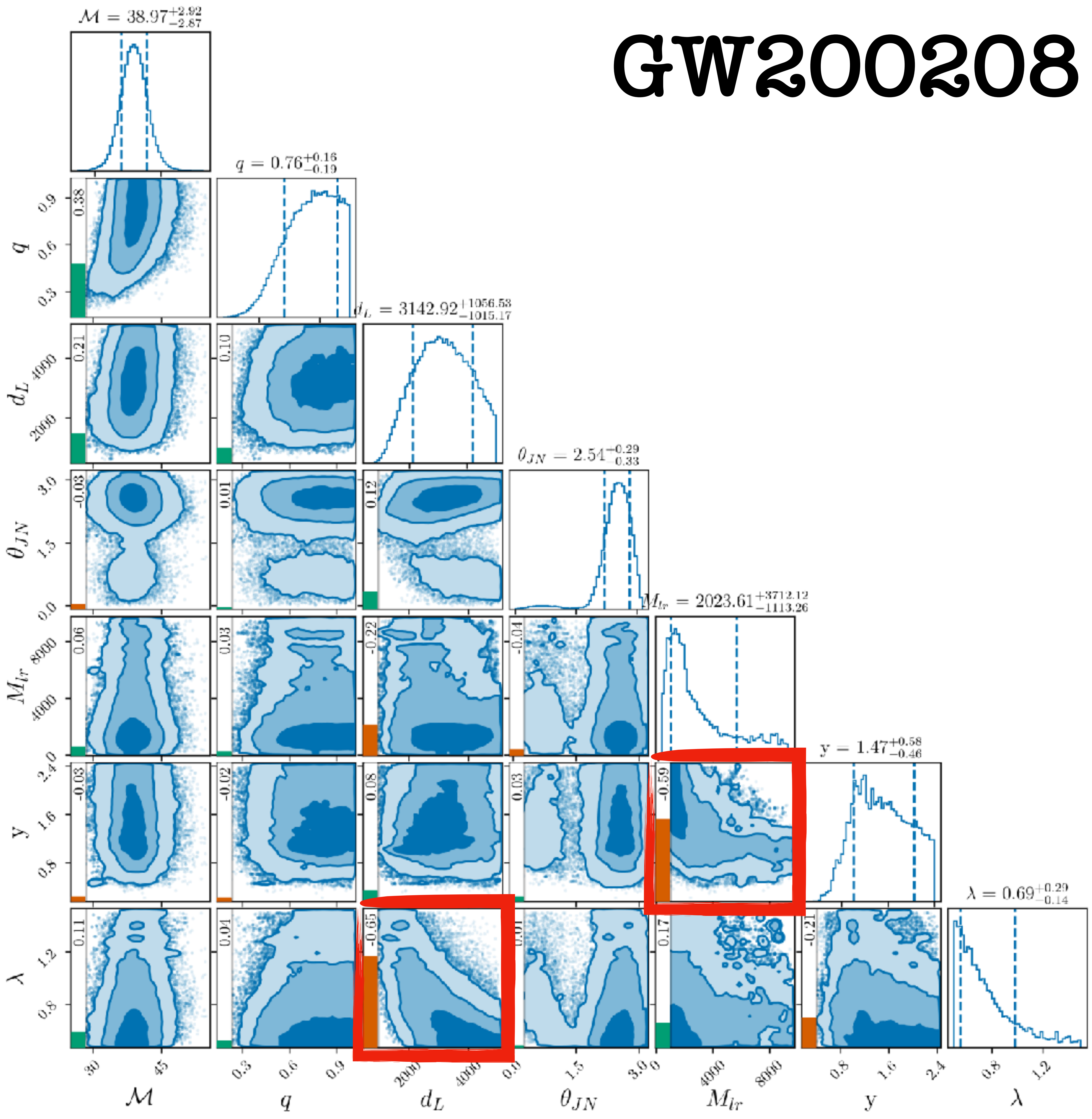
GW200208

no λ in PE
λ in PE
$\mathcal{M} = 38.97^{+2.90}_{-2.88}$
$q = 0.77^{+0.16}_{-0.19}$
$d_L = 2294.60^{+606.67}_{-556.89}$
$\theta_{JN} = 2.56^{+0.28}_{-0.34}$
$M_{lr} = 1693.47^{+1555.91}_{-922.60}$
$y = 1.47^{+0.49}_{-0.38}$
$\mathcal{M} = 38.97^{+2.92}_{-2.87}$
$q = 0.76^{+0.16}_{-0.19}$
$d_L = 3142.92^{+1056.53}_{-1015.17}$
$\theta_{JN} = 2.54^{+0.29}_{-0.33}$
$M_{lr} = 2023.61^{+3712.12}_{-1113.26}$
$y = 1.47^{+0.58}_{-0.46}$
$\lambda = 0.69^{+0.29}_{-0.14}$
$\mathcal{M}$
q
d_L
θ_{JN}
M_{lr}
y
λ

Conclusions 2/2

- The MSD is parametrised correctly
- For injections
  - y correlates far less with λ than $M_{l,r}$
  - considering λ increases $M_{l,r}$ errors
  - the behaviour of $M_{l,r}$ and y depend considerably on other parameters (d_L)
  - we are able to solve the degeneracy
- For a real*(istic)* event
  - low SNR complicates everything
  - degeneracy is not transmitted to λ
  - λ correlates much more with d_L than $M_{l,r}$ or y

GW200208



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