## Wave-optics in Gravitational Waves lensed events

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Based on arXiv:1911.11786 arXiv:2104.07055 arXiv:2111.01163







## Szczecin cosmology group

## Where is Szczecin?



## Cosmology group

## cosmo.usz.edu.pl







## **Gravitational Wave lensing**







## **Geometrical-optics vs wave-optics**



## LHS = $10^4 M_{\odot}$ $f \approx 10^2 \text{ Hz}$ $RHS = 10^3 M_{\odot}$ GO stands!



Geometrical-optics approximation breaks when

$$M_{3D,L} \le 10^5 M_{\odot} \left[ \frac{(1+z_L)f}{\text{Hz}} \right]^{-1}$$
$$f \cdot \Delta t \le 1$$

R.Takahashi,Astrophys.J.835,103(2017),arXiv:1606.00458 [astro-ph.CO





## GL of GW







characteristic strain  $[\sqrt{4f^2|\tilde{h}(f)|^2}]$ 

 $h(t) \cdot e^{-i2\pi ft} dt = \tilde{h}(f)$ 





## GL of GW



 $\tilde{h}(f) \cdot F(\theta_s, f) = \tilde{h}_L(f)$ 











- 0.0 strain -0.5
  - -1.0
  - -1.5
  - -2.0

 $\tilde{h}_L(f) \cdot e^{i2\pi ft} df = h_L(t)$ 

1e-21



## GL of GW





 $h_L(t)$  vs h(t)

## **Amplification Factor**

Geometrical Optics:

 $F(f) = \sum_{i} \sqrt{\mu^{(j)}} exp(2\pi i f \Delta t^{(j)} - i n^{(j)} \pi/2)$ 

Wave Optics:

 $-F(w, y) = -iwe^{iwy^2/2} \int_0^\infty dx x$ 

## $\tilde{h}(f) \cdot F(\theta_{s}, f) = \tilde{h}_{I}(f)$

$$J_0(wxy)\exp\left\{iw\left[\frac{1}{2}x^2-\Psi(x)\right]\right\}$$

$$v = \frac{1 + z_L}{c} \frac{D_S D_L \theta_E^2}{D_{LS}} 2\pi f$$

•  $x = |\overrightarrow{x}| = |\overrightarrow{\theta}/\overrightarrow{\theta}_E|$ 

•  $y = |\overrightarrow{y}| = |\overrightarrow{\theta_s}/\overrightarrow{\theta_E}|$ 

# High accuracy on $H_0$ constraints from gravitational wave lensing event

Based on arXiv:1911.11786 - Phys. Dark Univ. 28 (2020) 100517 with V. Salzano



## Cosmology

## $F(w, y) = -iwe^{iwy^2/2} \int_0^\infty dx \, x J_0(wxy) \exp\left\{iw\left[\frac{1}{2}x^2 - \Psi(x)\right]\right\}$



$$- z_L \frac{D_S D_L \theta_E^2}{D_{LS}} 2\pi f$$

$$c = |\vec{\theta} / \vec{\theta}_E|$$

$$| = |\vec{\theta} / \vec{\theta}_E|$$

GO:  $T_{EM}(x, y) = \frac{1}{2}(x - y)^2 - \psi(x)$ [Schneider, Ehlers, Falco 1992] WO:  $T_{GW}(w, y) = -\frac{i}{w} \ln\left(\frac{F(w, y)}{|F(w, y)|}\right)$ 



## the arrival time difference

## How? **EM-GW time-delay**

## $T_{\mathrm{EM},\pm-\mathrm{GW}}(y,w) = T_{\mathrm{EM},\pm}(y) - T_{\mathrm{GW}}(y,w)$

assuming a realistic observed model [Buote and Barth 2019]

2.

1.

## Lens Models

singular isothermal sphere (SIS)  $\rho(r) = \frac{\sigma_*^2 \ 1}{2\pi G \ r^2}$ with a stellar dispersion velocity  $\sigma_*^2 = 220$  km/s

## Navarro-Frenk-White (NFW)



## Methodology

- we calculate  $\Delta T_{EM-GW}$  fo  $\{\Omega_m, H_0\}$
- we assume an independer  $\Omega_m = 0.3061 \pm 0.0052$
- we infer the uncertainty on the time-delay uncertainty

## • we calculate $\Delta T_{EM-GW}$ for a large set of input parameters

## - we assume an independent prior on $\Omega_m$ from Planck,

## • we infer the uncertainty on $H_0$ by crossing the prior with

## Methodology





uncertainty on GW time-delay

 $\sigma_{\Delta T} = (2\pi f \rho^2)^{-1}$ 

where

[Huerta at el. 2015]





- state-of-the-art sample made of 65 pulsars observed with PTA
- an "optimistic" future sample of 1000 pulsars detected with SKA

ZS	$\mathcal{M}$ $(10^8 M_{\odot})$	T <sub>obs</sub> (yr)	σ <sub>rms</sub> (ns)	$\Delta \tau$ (yr)	N <sub>p</sub>	$\sigma_{\Delta T}$ (da
0.5	5	10	100	0.038	65 1000 65 1000	0.8 0.0 1.4 0.0

## Methodology

[Perera at el. 2019]

[Weltman at el. 2018]



## Results

## NFW lens - IPTA 65 pulsars array



## Results

## NFW lens - SKA 1000 pulsars array

	0.40	)					
g	0.38	; -					
	0.36						
	0.34						
	0.32						
	0.30						
	0.28	; -		0	F		
	0.26	, - , -	$z_s =$ y=	0.	.5 1		
	0.24		$\sigma_{\Delta t}$	= (	0.0	03	da
	0.22		$\sigma_{68} \ \sigma_{74}$	=	0.0	)6 )6	
	0.20	_ 60	62	2	64	(	56



## Results

$Z_S$		0.5				1	
$\sigma_{\Delta T}$ (days)	0.835		0.003		1.431		0.00
$H_0 \ ({\rm km} \ {\rm s}^{-1} \ {\rm Mpc}^{-1})$	68	74	68	74	68	74	68
$y \downarrow$	NFW - ACDM						
0	1.37	1.55	0.06	0.06	2.19	2.47	0.06
0.1	1.62	1.85	0.06	0.06	2.60	2.94	0.06
0.5	3.72	4.49	0.06	0.07	5.60	6.05	0.07
	NFW - quiessence						
0	14.50	15.80	12.10	14.20	14.70	16.20	12.60
0.1	14.60	16.00	12.60	14.20	15.00	16.70	12.60
0.5	16.50	18.20	13.10	14.30	16.90	19.30	12.70
	SIS - $\sigma_*=220~(\mathrm{km/s})$ - $\Lambda\mathrm{CDM}$						
0	2.80	3.15	0.06	0.07	4.56	5.13	0.07
0.1	3.06	3.43	0.06	0.07	5.03	5.63	0.07
1	>10	9.70	0.10	0.10	>10	>10	0.17
	SIS - $\sigma_* = 220$ km/s - quiessence						
0	15.80	17.30	12.90	14.20	16.10	18.70	12.70
0.1	16.00	17.60	12.90	14.20	16.40	19.00	12.70
1	>20.00	>20.00	13.20	14.40	>20.00	>20.00	12.80

 $H^{2}(z) = H_{0}^{2} \cdot \left[\Omega_{\gamma}(1+z)^{4} + \Omega_{m}(1+z)^{3} + \Omega_{\Lambda}(1+z)^{3(1+w)}\right]$ 



## **Conclusions** 1/3

- need for different measurement to decrease the error  $(\sigma \sim 1/\sqrt{n})$
- SKA will give decisive results

## • today observations could match current precision on $H_0$



## Based on <u>arXiv:2104.07055</u> - Phys. Rev. D 104, 023503 (2021) with J.M. Ezquiaga and V. Salzano





E. E. Falco, M. V. Gorenstein, and I. I. Shapiro, ApJ 289, L1 (1985)

• Scalings of lens mass:

$$- \kappa \to \kappa_{\lambda} = \lambda \kappa + (1 - \lambda)$$

• Scaling angles:



## **Mass Sheet Degeneracy**



## Why a problem?

- Observables are preserved!
- Problems: e.g. biased estimations of mass lens
- Biased estimation of cosmological parameter, e.g.  $H_0$

## MSD

## Can we solve it?

- EM geometrical optics regime: multiple images; independent mass estimation of the lens (e.g. dynamics)
- EM wave optics regime: multiple lenses
- In GW lensing: 1 image and 1 lens can break MSD!



























## **Gravitational Lensing of Grav. Waves**

- $\tilde{h}(f) \cdot F(f, \theta_s) = \tilde{h}_L(f)$
- $F(w, y) = -iwe^{iwy^2/2} \int_{0}^{\infty} dx \, x J_0(w.$
- Where:
- $w = \frac{1 + z_L}{c} \frac{D_S D_L \theta_E^2}{D_{LS}} 2\pi f$ •  $x = |\overrightarrow{x}| = |\overrightarrow{\theta}/\overrightarrow{\theta}_E|$ •  $y = |\overrightarrow{y}| = |\overrightarrow{\theta_s}/\overrightarrow{\theta_E}| \longrightarrow v_2$

## NB: spherical symmetry!

$$exy)\exp\left\{iw\left[\frac{1}{2}x^2-\Psi(x)\right]\right\}\longrightarrow F$$

## T. T. Nakamura and S. Deguchi, Progress of TheoreticalPhysics Supplement133, 137 (1999).

- $J_0$  Bessel function of 0-th order
- $\Psi$  dimensionless effective lensing potential







## Lensed waveforms under mass-sheet transformation en sel restante se restant and the second second and the second second second second second second second second Winding an and store and a straight the set and store and a straight the set and a straight the set and the store and the set and the set

Qualitative analysis



## Lensed GWs 3 regimes

## Geometrical Optics

•  $f \cdot \Delta t \gg 1$ •  $M_L > 10^5 [(1 + z_L)f]^{-1}$ 

strain

## $M_S = 60 \ M_{\odot} - z_S = 0.5$ $M_L = 10^4 \ M_{\odot} - z_L = 0.1 - y = 5$

R.Takahashi,Astrophys.J.835,103(2017),arXiv:1606.00458 [astro-ph.CO].





R.Takahashi,Astrophys.J.835,103(2017),arXiv:1606.00458 [astro-ph.CO].



## Lensed GWs



## Wave optics

## Lensed GWs Wave optics

q = 1

 $10^{-16}$  characteristic strain [ $\sqrt{4f^2}| ilde{h}(f)|^2$ ]  $10^{-17}$  $10^{-18}$ 

 $10^{1}$ 





 $M_L = 500 \ M_{\odot} \& y = 1 \& z_L = 0.01$ 

 $M_S = 100 \ M_{\odot} \& q = 1 \& z_S = 0.1$ 





 $M_L = 500 \ M_{\odot} \& y = 1 \& z_L = 0.01$ 

## Lensed GWs

## Interference regime

 $^{\circ}f\cdot\Delta t\approx1$ 



 $M_S = 100 \ M_{\odot} \& q = 1 \& z_S = 0.1$  $M_L = 500 \ M_{\odot} \& y = 1 \& z_L = 0.01$ 

## S/N - template matching The second second second in a static the Antonio and in the second second in and in the second second second second second second second second second s

Quantitative analysis

## Signal-to-Noise ratio

 $\rho = \frac{\left(s \mid h_T\right)}{\sqrt{(h_T \mid h_T)}} \approx \frac{\left(h \mid h_T\right)}{\sqrt{(h_T \mid h_T)}}$ 

M. Maggiore, Gravitational Waves: Volume 1: Theory and Experiments, Gravitational Waves (OUP Oxford, 2008)

• s(t) = h(t) + n(t)

• Inner product:

$$(a \mid b) = 4 \operatorname{Re} \left[ \int_{0}^{\infty} \frac{\tilde{a}(f) \cdot \tilde{b}^{*}(f)}{S_{n}(f)} df \right]$$

•  $S_n(f)$  - (single-sided) power spectral density (L1-O3-LIGO)

Confidence region:  $\Delta \chi^2 \approx 2 \rho_{opt}^2$   $1 - \frac{\rho}{1}$ 

 $3\sigma \rightarrow \Delta \chi^2 \approx 11.8$ 







 $\lambda_{min} = 0.93$ 

S/



 $\lambda_{min} = 0.99$ 

0.85 - 0.54 0.54 0.57 0.63 0.71 0.80 0.88 0.93 0.95 0.94 0.9 - 0.59 0.61 0.67 0.74 0.83 0.91 0.96 0.98 0.96 0.91 **0.95** - 0.65 0.69 0.76 0.84 0.92 **0.98 1.00 0.97** 0.92 0.85 **1.0** - 0.70 0.76 0.84 0.92 **0.98 1.00 0.98** 0.92 0.83 0.76 **1.05** - 0.76 0.83 0.90 **0.97 1.00 0.98** 0.91 0.82 0.74 0.67  $\rightarrow$ **1.1** - 0.81 0.88 0.95 0.99 0.98 0.91 0.82 0.72 0.64 0.59 **1.15** 0.86 0.93 0.97 0.98 0.92 0.82 0.71 0.61 0.55 0.50 **1.2** - 0.90 0.95 0.97 0.93 0.83 0.71 0.60 0.52 0.46 0.40 **1.25** - 0.93 0.96 0.94 0.85 0.73 0.60 0.50 0.43 0.36 0.28 1.3 - 0.94 0.95 0.88 0.76 0.62 0.50 0.42 0.34 0.25 0.15  $M_L [10^2 M_{\odot}]$ 

 $\lambda = 1$ 

1.0

0.8

-0.6

-0.4

0.2

0.0

1.0

0.8

0.6

0.4

-0.2

0.0

0.95 - 0.901 0.919 0.936 0.952 0.965 0.976 0.984 0.990 0.994 0.994 0.96 0.920 0.938 0.953 0.966 0.978 0.986 0.992 0.996 0.996 0.995 **0.97** - 0.938 0.954 0.967 0.978 0.987 0.994 0.997 **0.998** 0.996 0.992 **0.98** - 0.954 0.967 0.979 0.988 0.994 **0.998 0.999** 0.997 0.993 0.986 **0.99** - 0.967 0.979 0.988 0.995 **0.999 1.000 0.998** 0.994 0.987 0.978 **1.0** - 0.978 0.987 0.994 0.999 1.000 0.999 0.994 0.988 0.979 0.967 **1.01** - 0.986 0.994 0.998 1.000 0.999 0.995 0.988 0.979 0.967 0.954 **1.02** 0.992 0.997 **0.999 0.998** 0.995 0.988 0.979 0.968 0.954 0.939 **1.03** 0.996 0.998 0.998 0.994 0.988 0.979 0.968 0.954 0.939 0.922 **1.04** - 0.997 0.997 0.994 0.988 0.979 0.968 0.954 0.939 0.921 0.903 4. 2° 

## $t_{max} = 1.03$



тах

5.00 5.2 5.2 5.2 5.2 5.2 5.2 5.2 5.2 5.0  $M_L [10^2 M_{\odot}]$ 







opt

S/N

1.02 -

Q.9

Interference regime



 $\Lambda_{max} = 1.03$ 

0.9 - 0.570 0.565 0.580 0.638 0.735 0.843 0.928 0.965 0.949 0.894 **1.0** - 0.644 0.662 0.722 0.819 0.920 0.986 0.991 0.937 0.852 0.777 **1.1** - 0.713 0.766 0.856 0.948 0.997 0.974 0.885 0.777 0.695 0.651 **1.2** - 0.786 0.865 0.949 0.988 0.945 0.830 0.704 0.614 0.557 0.495 **1.3** - 0.858 0.938 0.977 0.925 0.792 0.650 0.549 0.474 0.382 0.258 **1.4** - 0.919 0.967 0.922 0.781 0.622 0.507 0.413 0.297 0.151 0.004 **1.5** - 0.955 0.933 0.798 0.623 0.490 0.381 0.246 0.084 -0.071 -0.202 **1.6** - 0.950 0.840 0.655 0.501 0.378 0.230 0.055 -0.109-0.245-0.357 **1.7** - 0.896 0.717 0.540 0.403 0.247 0.060 -0.115 -0.260 -0.377 -0.461  $M_L [10^2 M_{\odot}]$ 0.98 -0.920 0.934 0.948 0.960 0.971 0.980 0.987 0.993 0.996 0.997 0.99 -0.937 0.950 0.962 0.973 0.982 0.989 0.994 0.997 0.999 0.998 **1.0** - 0.952 0.964 0.974 0.983 0.990 0.995 0.998 1.000 0.999 0.996 **1.01** - 0.965 0.975 0.984 0.991 0.996 **0.999 1.000 0.999** 0.996 0.991 **1.02** - 0.976 0.984 0.991 0.996 **0.999 1.000 0.999** 0.996 0.990 0.983 **1.03** - 0.984 0.991 0.996 **0.999 1.000 0.998** 0.995 0.990 0.983 0.974 **1.04** - 0.991 0.996 **0.998 0.999** 0.998 0.994 0.989 0.981 0.972 0.962 **1.05** - 0.995 0.998 **0.998** 0.997 0.993 0.988 0.980 0.971 0.960 0.948 1.06 - 0.997 0.997 0.996 0.992 0.987 0.979 0.969 0.958 0.946 0.932

 $M_L$  [10<sup>2</sup>  $M_{\odot}$ ]

P. Schneider and D. Sluse, Astron. Astrophys.559, A37(2013), arXiv:1306.0901 [astro-ph.CO]





## **Conclusions** 2/3

- 1. We analysed how MSD behave in GW lensing 2. In the GO regime it can not be broken
- 3. In WO can be broken in some cases
- 4. In interference regime is broken

5. How well it is broken depends on the strength of the signal and sensitivity of detectors. Nowadays we might have up to  $\Delta y \approx 5\,\%\,$  and  $\Delta M \approx 6\,\%\,$ 



# High precision lens modelling

Based on <u>arXiv:2111.01163</u> with D.F. Mota and V. Salzano

## **Gravitational Wave lensing**



## High precision lens modelling Lens mass profile



## High precision lens modelling Lensed waveforms





 $M_L(r_c) = 10^9 M_{\odot}$ 

 $z_{L} = 0.15$ 



## **Unlensed vs lensed** Lensed waveforms



We would need  $\rho \approx 4000$ 

## **Unlensed vs lensed** Lensed waveforms





- $\rho \approx 100$
- $\frac{\rho}{---} = 0.9869$
- $\Delta \chi^2 \approx 11.8$
- $\frac{\rho}{\rho_{opt}} = 0.9994$

- SIS / gNFW<sub> $\gamma=2$ </sub>
- 2 free parameters
  - $3\sigma$  threshold









We would need  $\rho \approx 2200$ 







## 1. Lensed events can be misinterpreted by unlensed one

- 2. Studying the phase of the signal is more effective than matched filtering
- 3. We can differentiate between lens models
- 4. Differentiating between models is useful to study dark matter/dark energy content